219B – Exercise on Present Bias and Retirement Savings

Question #1

In this Question we consider the impact of self-control problems on investment in retirement savings with a similar setting to DellaVigna (2018). Consider a present-biased individual, characterized by time preference parameters $\delta, \beta, \hat{\beta}$, that is considering when (and whether) to call the Human Resources Department and sign up for a 401(k) plan. Saving for retirement involves an immediate effort cost of $-k < 0$ at time $t = 0$ (the present), but allows the individual to set aside s dollars in savings each day, starting from the present ($t = 0$) all the way to the period before retirement ($T - 1$). The savings are matched by the employer with a match of net-rate μ, and in addition accumulate net interest per-period r. Hence, s dollars set aside today become $s(1 + \mu)(1 + r)^t$ dollars in t periods (days). All savings are paid as a lump sum in retirement at period T, which we assume to be far enough. The individual has to choose when to undertake the investment activity, that is, at t, at $t + 1$, at $t + 2$, etc. (The individual can also decide not to do it). Assume that k, r, μ and s are deterministic, and that consumption utility is linear ($u(x) = x$) and identical in all time periods.

Two important points/assumptions that are relevant to the calculations below: (i) Once the individuals pays the cost $-k$ and starts saving for retirement, s dollars of savings are set aside in each period t up to $T - 1$ (the day before retirement); (ii) The cost of saving s in period t is foregoing s dollars of consumption in that period, and in all following periods up to $T - 1$.

a) Write down the utility of investing immediately, U_0, from the perspective of the time-0 self, compared to the utility of never investing.

b) Write down the utility of investing at a future period U_τ for $\tau > 0$, still from the perspective of the time-0 self.

c) From now on assume that $\delta = \frac{1}{1+r}$, simplify the expressions in the points above accordingly.

d) Consider first a time-consistent individual ($\beta = \hat{\beta} = 1$) and solve for the optimal timing of the investment decision. Show that the optimal solution takes the form of a threshold rule in k as a function of μ, δ, T, and s.

e) What are the qualitative comparative statics with respect to μ and s? Also, how does the threshold vary with T (the distance from retirement)? Discuss the intuition behind these comparative statics.

f) Consider then a sophisticated present-biased individual ($\beta = \hat{\beta} < 1$). There actually are multiple equilibria on the period t at which the sophisticated agent will invest (if at all) (O'Donoghue and Rabin, 2001). Despite this multiplicity, it is possible to provide a bound on the delay in investing. Namely, show that a sophisticated agent will wait for at most τ days to invest if the cost of investing k satisfies (for δ close to 1) [You will need a Taylor expansion of $1 - \delta^T$ for δ going to 1: $1 - \delta^T \simeq (1 - \delta) T$]:

$$k \lessapprox s \left[\frac{\beta}{1 - \beta} \mu \tau - 1 \right] \tag{1}$$

g) Consider now a fully naive present-biased individual ($\beta < \hat{\beta} = 1$). As of time $t = 0$, under what conditions does the individual expect to invest tomorrow (at $t = 1$)? Under what conditions does the individual expect to never invest?

h) Comparing now the utility from investing today and the utility of investing tomorrow, show that the fully naive present-biased individual invests immediately, that is, at time $t = 0$ (and otherwise never invests) if and only if

$$k \leq \frac{s \left[\beta(1 + \mu) - 1 \right]}{1 - \beta\delta}$$

i) What is the range of costs such that a naive agent expects to invest, but does not invest? We call that *procrastination* as opposed to delay.

j) In this simple model, what happens if there is no match on the 401(k)s, that is, if $\mu = 0$?

k) Even in absence of any match, which other reasons to save could individuals have? Without solving the model under these alternative assumptions, think about some of the simplifying assumptions we made initially.

Question #2

We now expand on Question 1 but we remove the assumption that all variables are deterministic. In particular, we still assume that r, μ and s are still deterministic, but we allow for a stochastic cost of time k. This could capture for example the fact that an employee is busier on some days compared to other days. In particular, we assume that k is drawn with i.i.d. draws in each period t from a distribution F. We consider the decision of an individual at time $t = 0$.

a) Show that the decision rule for an exponential agent will be of the type: Invest today if $k < k_t^e$ where k_t^e is defined by

$$-k_t^e + s\mu\frac{1-\delta^{T-t}}{1-\delta} = \delta V_{t+1}^e \tag{2}$$

and V_{t+1}^e is the value function of the exponential agent at period $t+1$. Notice that in particular in period $t = 0$ this becomes

$$-k_0^e + s\mu\frac{1-\delta^{T}}{1-\delta} = \delta V_1^e, \tag{3}$$

but we need to solve for k_t^e for all periods $t = 0, 1, ..., T-1$, since the threshold k_t^e is time-varying (given that the retirement period gets closer).

b) This problem can be solved iterating by backwards induction, starting from the last decision period, which is day $T-1$, the last investment period before retirement. Solve for k_{T-1}^e and then iterate to solve fork_{T-2}^e. [We will find k_t^e numerically in the next sections]

c) Show that the rule for a fully naive present-biased agent is: Invest today if $k < k_t^n$ where k_t^n is defined by

$$-k_t^n + s\left(\beta\left(1+\mu\right)-1\right) + \beta s\mu\frac{\delta-\delta^{T-t}}{1-\delta} = \beta\delta V_{t+1}^e \tag{4}$$

and V^e_{t+1} is the value function of the exponential agent. Compare (2) and (4) and obtain a relationship between k^n and k^e. Explain that relationship intuitively.

d) Follow the code on the file "`stochastic_saving_code.R`" to solve the dynamic programming problem of the exponential agent. Assume the time period is one day and run the code for values $\delta = 0.9999, \mu = 0.2, s = \$10, T = 3000$, and uniform cost between 0 and 50. Explain qualitatively which assumptions we are making for the parameters.

e) Plot the pattern of k^e_t for t that goes from 0 to 3,000. Zoom in on periods 2,900 to 3,000 (the last 100 days before retirement). Explain the patterns you find. Compute the expected delay (in number of days) until signing up for retirement for a person starting in period 0.

f) Redo the same, but assume $\mu = 0$ (no match). Explain the patterns you find.

g) Redo the same, go back to assume $\mu = 0.2$, but change $s = 1$ (less money to save). Explain the patterns you find.

h) Now keep $\mu = 0.2$ and $T = 3000$, and choose the other parameter values so as to get longer average delays, while still picking reasonable parameters. (Defend your choices of what is plausible.) How long of a delay do you get?

i) Based on the above, expand the code to also solve the problem of the naive agent.
Hint 1: in equation 4 the naive uses the value function of the exponential.
Hint 2: you also found a relationship between k^n and k^e.

j) Redo the simulations in points (4-5) assuming $\beta = .8$, computing in particular the expected delay. In the model with stochastic k, do naives delay forever (as we found they do in the deterministic case for plausible values)?

k) Now revise the previous point, but assume a uniform cost distribution between 45 and 55. Compute the expected delay for a naive with $\beta = .8$ and an exponential, for otherwise the same parameters. Do naives delay forever now?

l) Discuss the role of the distribution of costs k.

Question #3

In this Question we apply the results of Question 1 to default effects in 401(k) choice (Madrian and Shea, 2001; Choi et al., 2006; Carroll et al., 2009—all in the reading list).

a) Consider the attached Table 1 (consider only companies B, C, D, and H) and Figures 1a-1d from the survey paper "Saving for Retirement on The Path of Least Resistance" (Choi et al., 2006). Choi et al. report the result of changes in default for 401(k) investments in 4 companies. The change is of a similar type as in Madrian and Shea (2001) Comment on the findings in the Figures. Describe the effect of the change in default.

b) To what extent does this evidence go beyond the evidence in Madrian and Shea (2001)? Cite at least one concern with the evidence in the Madrian and Shea (2001) paper that the evidence in Choi et al. (2006) addresses.

c) Now we go back to the answers in Questions 1 and 2 and calibrate them to address the evidence in Madrian and Shea (2001) and Choi et al. (2006). As in Question 1, consider a new employee in a company without automatic enrollment (that is, the default is no investment). In the opt-in regime (OLD regime in Madrian and Shea), the employee can pay an effort cost $k > 0$ and invest in the 401(k), thereafter reaping saves s in every subsequent day. In the opt-out regime, assume that the effort cost k to invest is negative, $k < 0$, given that investing is the default. Finally, for the active-choice case, assume $k = 0$, given that the person has to pay a cost either way, whether investing or not, so the cost cancels. Provide reasonable values for the key parameters for the deterministic case for a young workers (T is large) with average earnings? Use the info in Madrian and Shea (2001) as possible. Justify the assumptions you make. (Remember: s and δ are on a *daily* scale).

d) For these parameters is it likely that a default change would cause much difference in the average days to investment for an exponential individual? Discuss based on your answers to Question 1 (deterministic case) and Question 2 (stochastic case). Does it look like the exponential case can fit the observed default effects?

e) Now use the same parameters, but allow for a sophisticated worker with $\beta = 0.8$. Discuss based on your answers to Question 1 (deterministic case), since we did not do the sophisticated case for Question 2 (stochastic case). Does it look like the sophisticated case can fit the observed default effects?

f) Now use the same parameters, but allow for a naive worker with $\beta = 0.8$. Discuss based on your answers to Question 1 (deterministic case) and Question 2 (stochastic case). Does it look like the naive case can fit the observed default effects?

g) In light of these points, which calibrated model fits the data better? Why? Does it matter if the costs are stochastic?

h) Can you envision ways to further enrich or change the above model to make it more realistic?

Figures from Choi et al. (2006)

TABLE 1. Companies and Their 401(k) Plan Changes or Other Interventions

Company	Industry	Size[a]	Plan Change / Intervention	Date of Change/ Intervention
A	Food	10,000	Savings survey	January 2001
B	Office equipment	30,000	Adopted automatic enrollment	January 1997
			Eliminated automatic enrollment	January 2001
C	Insurance	30,000	Adopted automatic enrollment	April 1998
			Financial education seminars	January-December 2000
			Changed automatic enrollment defaults	May 2001
D	Food	20,000	Adopted automatic enrollment	January 1998
			Increased default contribution rate	January 2001
E	Utility	10,000	Increased match threshold	January 1997
F	Consumer packaged goods	40,000	Changed eligibility	July 1998
			Instituted employer match	October 2000
G	Insurance	50,000	Changed eligibility	January 1997
H	Manufacturing		Adopted automatic enrollment	January 2001
I	Retail	130,000	None	NA
J	Financial Services	50,000	None	NA
K	Pharmaceutical	10,000	Changed eligibility	January 1996

[a] Number of employees (rounded to the nearest 10,000) on December 31, 1998 (Company K), December 31, 2000 (Companies A, B, D, E, F, G, I and J), June 30, 2000 (Company C) or December 31, 2001 (Company H).

Figure 1A. 401(k) Participation by Tenure: Company B

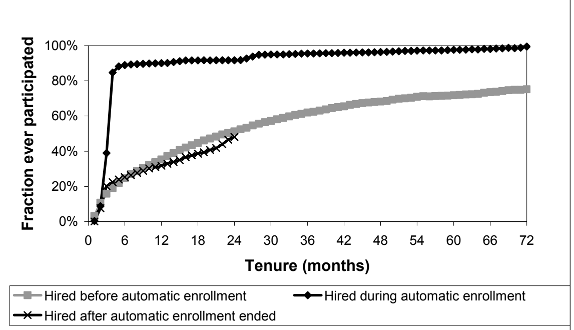


Figure 1C. 401(k) Participation by Tenure for Employees Aged 40+ at Hire: Company D

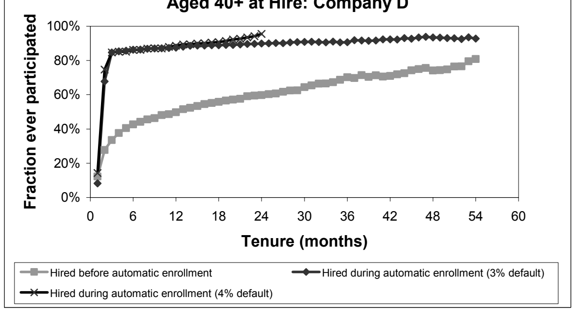


Figure 1B. 401(k) Participation by Tenure: Company C

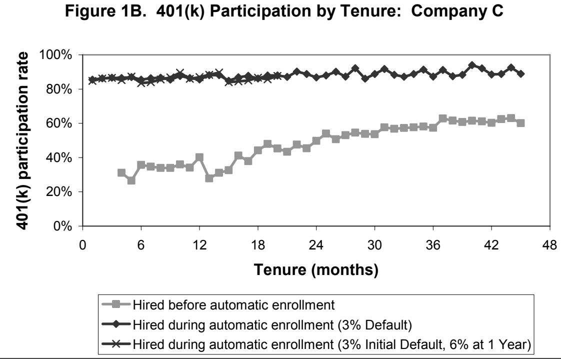
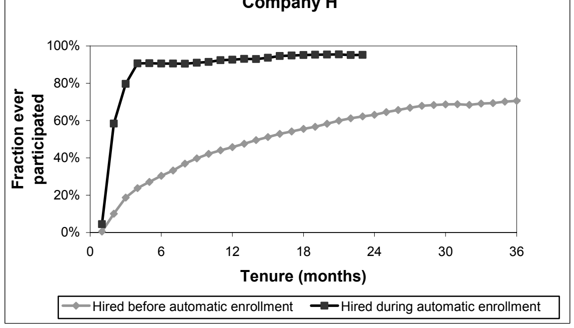


Figure 1D. 401(k) Participation by Tenure: Company H



References

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