UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2019
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Problem Set 5
Due at the start of class, Thursday, October 10

1. (Natural resources in a model of knowledge accumulation.) Consider the following variant of the model of knowledge accumulation and growth in Section 3.2 of *Advanced Macroeconomics*. $R(t)$ denotes use of natural resources at time t, and a_R denotes the fraction of those resources that are used in the R&D sector. The rest of the notation is standard.

$$Y(t) = A(t)[(1 - a_L)L(t)]^{\beta}[(1 - a_R)R(t)]^{1-\beta}, \quad 0 < a_L < 1, 0 < a_R < 1, 0 < \beta < 1,$$

$$\dot{L}(t) = nL(t), \quad n > 0,$$

$$\dot{R}(t) = -\mu R(t), \quad \mu > 0,$$

$$\dot{A}(t) = B[a_L L(t)]^{\gamma}[a_R R(t)]^{\varphi} A(t)^{\theta}, \quad B > 0, \gamma > 0, \varphi > 0.$$

Assume $\theta < 1$. $A(0)$, $L(0)$, and $R(0)$ are all strictly positive.

a. Define $g_A(t) \equiv \dot{A}(t)/A(t)$. Derive an expression for $\dot{g_A}(t)$ in terms of $g_A(t)$ and the parameters.

b. Sketch the function you found in part (a). For what values of g_A is $\dot{g_A} = 0$? For what parameter values and/or initial conditions does g_A converge to each of these values?

c. What is the growth rate of output per person on the balanced growth path as a function of the parameter values and/or initial conditions?

2. Consider the production function of the Paul Romer model, and suppose that a firm changes from using inputs in equal amounts (for example, $L(i) = b$ for $0 \le i \le A$, where b and A are both positive) to using inputs in unequal amounts with no change in the total quantity of inputs (for example, $L(i) = 2bi/A$ for $0 \le i \le A$). Then the firm's output will:

A. Increase.
B. Decrease.
C. Stay the same.
D. It is not possible to tell.

3. Romer, Problem 4.3.

4. (Notes: (i) This problem is related to material that will be covered in lecture on Oct. 8. (ii) It is based on a problem on last year's final. (iii) In answering it, you can assume that the regressions being described do not suffer from omitted variable bias.)

Although it is not good practice, some papers report only point estimates and t-statistics and do not report standard errors. You are reading a paper that examines the impact of average years of education on income per capita. Specifically, the researchers estimate regressions of the form,

$$\ln\left(\frac{Y_i}{POP_i}\right) = a + bE_i + u_i,$$

where i indexes counties, E_i is the average number of years of education of workers in country i, and Y_i/POP_i is income per person in country i.

a. Here are 4 possible ***results*** the paper might report about the impact of schooling on log income per capita:

	Point estimate	t-statistic
#1	0.001	10.0
#2	−0.0002	−0.1
#3	0.4	0.5
#4	0.3	6.0

(Thus, for example, an entry of 0.4 in the first column corresponds to an estimate that an increase of 1 year in average years of schooling causes log income per capita to be higher by 0.4 than it would have been otherwise.) And here are 6 possible ***interpretations*** of the regression results:

i. Strong evidence that increased schooling has a large positive effect on income per capita.
ii. Moderate evidence that increased schooling has no effect whatsoever on income per capita.
iii. Little useful evidence about the effects of increased schooling on income per capita.
iv. Strong evidence that increased schooling has a positive effect on income per capita—but that the effect is small.
v. Moderate evidence that increased schooling has large negative effect on income per capita.
vi. Strong evidence that the effects of increased schooling on income per capita are not large.

For each ***result***, which ***interpretation*** is the most appropriate? Be sure to explain your answers.

b. In the regression described above, how would you interpret a point estimate of 0.4 with a t-statistic of 1.8? How would you interpret a point estimate of 0.4 with a t-statistic of 2.1? (In each case, a sentence of interpretation, together with a brief explanation of why that interpretation is appropriate, is all that is expected.)

EXTRA PROBLEMS (NOT TO BE HANDED IN; COMPLETE ANSWERS MAY NOT BE PROVIDED)

5. Consider an economy described by: $\dot{B}(t) = bB(t)$, $\dot{D}(t) = d[cB(t)]^{\omega}D(t)^{\mu}$, $J(t) = [(1-c)B(t)]D(t)$, with $b > 0$, $d > 0$, $0 < c < 1$, $\omega > 0$, $B(0) > 0$, and $D(0) > 0$. This economy will converge to a balanced growth path if and only if:

A. $\mu < 1$.
B. $\mu \leq 1$.
C. $\omega < 1$.
D. $\omega \leq 1$.

6. Romer, Problem 3.8.

7. Romer, Problem 3.11.

8. Romer, Problem 3.14.

9. Romer, Problem 4.4.

10. Romer, Problem 4.9.

11-16. Romer, Problems 3.2, 3.7, 3.12, 3.13, 4.1, 4.8.