UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2019
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# Problem Set 1

Due at the start of lecture, Tuesday, September 10

Problem set ground rules:

- You may work with classmates, but: (1) You must try the problems on your own first; (2) You must write up your answers yourself, in your own words; (3) If you work with others, you should thank them in an acknowledgment footnote at the start of your problem set.
- Looking at already written answers to the problems in any form is not permitted and would be considered academic misconduct.

1. Consider an economy described by the Solow model where initially capital per unit of effective labor, $k$, is less than its balanced-growth-path value. A permanent increase in the depreciation rate, $\delta$, will cause output per worker to be:

   A. Lower than it would have been otherwise during the transition to the balanced growth path.
   B. Lower than it would have been otherwise when the economy reaches its balanced growth path.
   C. (A) and (B).
   D. None of the above.

2. A first-order Taylor expansion of the equation for $\dot{k}(t)$ in the Solow model around the balanced-growth-path value of $k$, $k^*$, can be written as:

   A. $\dot{k}(t) \cong [sf'(k^*) - (n+g+\delta)]\left[\frac{dk(t)}{dt}\right]$.
   B. $\dot{k}(t) \cong [1 - (n+g+\delta)]\left[\frac{dk(t)}{dt}\right]$.
   C. $\dot{k}(t) \cong [sf'(k(t)) - (n+g+\delta)k(t)] + [sf'(k^*) - (n+g+\delta)][k(t) - k^*]$.
   D. $\dot{k}(t) \cong [\frac{sf'(k^*)k^*}{f(k^*)} - \frac{(n+g+\delta)k^*}{f(k^*)}]\frac{f(k^*)}{k^*}[k(t) - k^*]$.

3. Romer, Problem 1.9.

4. Romer, Problem 1.10.

5. Consider the Solow growth model. Find an expression for the elasticity of the balanced-growth-path level of output per unit of effective labor with respect to n+g+δ. Simplify your expression as much as possible.

6. (Hicks meets Solow.) Consider the Solow model with one change: the production function is Y = AF(K,L). All other assumptions of the model are unchanged. (The type of technical change assumed in the Solow model is known as labor-augmenting or Harrod-neutral technical change; the form of technical change assumed in this problem is called Hicks-neutral.)

   a. Show what happens if we try to derive a balanced growth path like the one derived in class.

   b. What can you say in the special case $F(K,L) = K^{\alpha}L^{1-\alpha}$, $0 < \alpha < 1$?

EXTRA PROBLEMS (NOT TO BE HANDED IN; COMPLETE ANSWERS MAY NOT BE PROVIDED)

7. Describe how, if at all, each of the following developments affects the break-even and actual investment lines in our basic diagram for the Solow model:

a. The rate of population growth falls.

b. The rate of technological progress rises.

c. The production function is Cobb-Douglas, $F(K,AL) = K^{\alpha}(AL)^{1-\alpha}$, and capital's share, α, rises.

d. Workers exert more effort, so that output per unit of effective labor for a given value of capital per unit of effective labor is higher than before.

8. (This is from the 2015 midterm.) In the Solow model with all of our usual assumptions, except that $n + g + \delta = 0$:

A. $k$ would converge to its unique balanced growth path value of 0.
B. $k$ would converge to some strictly positive, finite value.
C. $k$ would grow without bound.
D. It is not possible to determine the behavior of $k$.

9. Consider an economy described by the Solow model. Assume that initially capital and output per unit of effective labor are *less* than their balanced-growth-path values. Now suppose that in this situation, the saving rate rises permanently.

Sketch the resulting path of the log of output per worker and what that path would have been if the saving rate had not changed. Explain your answer.

10. (Problem 4, cont.) Romer, Problem 1.11.

11. (This is from the 2016 midterm.) Consider the following variant of the Solow model. There are two types of capital, 1 and 2. The production function is $Y(t) = F\big(K_1(t), K_2(t), A(t)L(t)\big)$, with constant returns to scale in the three arguments. Fraction $s_i$ of output is devoted to investment in capital of type $i$, which depreciates at rate $\delta_i$. Thus, $\dot{K}_1(t) = s_1 Y(t) - \delta_1 K_1(t)$ and $\dot{K}_2(t) = s_2 Y(t) - \delta_2 K_2(t)$. The other assumptions are standard: $\dot{L}(t)/L(t) = n$, $\dot{A}(t)/A(t) = g$, and $L(0)$, $A(0)$, $K_1(0)$, and $K_2(0)$ are all strictly positive. Assume $n + g + \delta_1 > 0$, $n + g + \delta_2 > 0$, $s_1 > 0, s_2 > 0, s_1 + s_2 \leq 1$.

a. Define $y \equiv Y/AL,\ k_1 \equiv K_1/AL,\ k_2 \equiv K_2/AL$. Show that one can write $y$ as a function of $k_1$ and $k_2$ (analogous to the fact the assumptions of the conventional Solow model allow us to write $y = f(k)$).

b. Derive expressions for $\dot{k}_1(t)$ and $\dot{k}_2(t)$ analogous to the Solow equation, $\dot{k}(t) = sf\big(k(t)\big) - (n + g + \delta)k(t)$. (That is, derive expressions for $\dot{k}_1(t)$ and $\dot{k}_2(t)$ in terms of $k_1(t)$, $k_2(t)$, the exogenous parameters, and the function you derived in part (a).)

c. Now assume that the production function is Cobb-Douglas:

$$Y(t) = K_1(t)^{\alpha} K_2(t)^{\beta} [A(t)L(t)]^{1-\alpha-\beta}, \quad \alpha > 0, \beta > 0, \alpha + \beta < 1.$$

In $(k_1,k_2)$ space, sketch the set of points such that $\dot{k}_1 = 0$, and explain your reasoning. Sketch the set of points such that $\dot{k}_2 = 0$.

d. Discuss the dynamics of $k_1$ and $k_2$ implied by your analysis in (c). (For example, is there a unique $(k_1^*, k_2^*)$ that the economy converges to regardless of its initial situation? Are there cases where $k_1$ and/or $k_2$ grow without bound? Etc.)