

LECTURE 17

April 2, 2019

CAPITAL AND INTEREST

I. OVERVIEW

- A. Our aggregate production function framework
- B. The role of capital in growth
- C. Terminology: capital versus investment
- D. Where we are headed

II. RENTAL MARKET FOR CAPITAL

- A. Profit maximization and the demand for rental capital
- B. Supply and equilibrium
- C. Complications when we think about a firm buying rather than renting capital

III. PRESENT VALUE

- A. Time preference and definition of present value
- B. Present value of a single payment to be received in the future
- C. Present value of a stream of payments to be received in the future

IV. PURCHASING CAPITAL AND THE INVESTMENT DEMAND CURVE

- A. Profit maximization and a firm's decision about how many machines to buy
- B. Investment demand curve
- C. The real interest rate and the investment demand curve
 - 1. The distinction between the nominal and real interest rate
 - 2. Why investment demand depends on the real interest rate
- D. Shifts in the investment demand curve

Economics 2
Spring 2019

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LECTURE 18

Capital and Interest



April 2, 2019

Announcements

- You should have handed in Problem Set 4.
- Midterm 2:
 - Tuesday, April 9th, 2:10–3:30.
 - You do ***not*** need a blue book.

Announcements (continued)

- **Midterm 2 Logistics:**
 - If your GSI is Todd Messer (Sections 101 and 102), go to 60 Barrows.
 - If your GSI is Priscila de Oliveira (Sections 103 and 104), go to 3108 Etcheverry.
 - If your GSI is Vitaliia Yaremko (Sections 111 and 114), go to 170 Barrows.
 - DSP students: You should have received an email from the head GSI (Todd Messer). If you haven't, please contact him (messertodd@berkeley.edu).
 - Everyone else come to the usual room (2050 VLSB).

Announcements (continued)

- **Midterm 2 Format:** Similar to Midterm 1.
- **Midterm 2 Coverage:**
 - Everything up to and including lecture on Thursday, April 4 (Saving and Investment in the Long Run).
 - Lecture, section, textbook, and additional readings.
 - There will be no questions ***solely*** about material from before Midterm 1.

Announcements (continued)

- Hints for Studying:
 - Start now!
 - Review lecture notes and slides; study problem set suggested answers.
 - Pose yourself problems.
 - Do the sample midterm by yourself.

Announcements (continued)

- Places to Get Help:
 - Sample midterm.
 - Professor and GSI office hours.
 - Review session: Friday, April 5, 6–8 p.m. in the usual lecture room (2050 VLSB).

I. OVERVIEW

Aggregate Production Function

$$(1) \quad \frac{Y^*}{POP} = \frac{Y^*}{N^*} \cdot \frac{N^*}{POP}$$

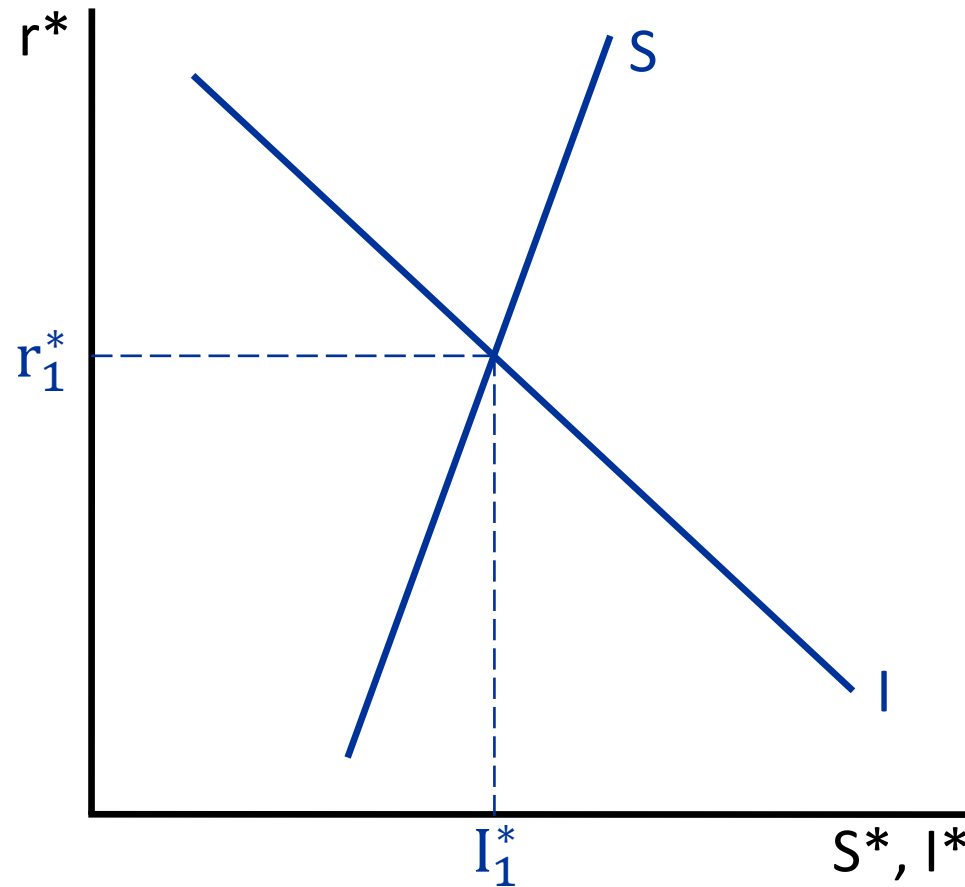
$$(2) \quad \frac{Y^*}{N^*} = f\left(\frac{K^*}{N^*}, T\right)$$

$$(3) \quad \frac{Y^*}{POP} = f\left(\frac{K^*}{N^*}, T\right) \cdot \frac{N^*}{POP}$$

Capital and Investment

- **Capital:** The accumulated ***stock*** of aids to the production process that were created in the past.
- **Investment:**
 - ***Changes*** in the capital stock.
 - That is, the construction or purchases of ***new*** machines and structures.

Where We're Headed: The Long-Run Saving and Investment Diagram



Here S is saving, I is investment, and r is the real interest rate (and $*$ denotes a long-run value).

Other Reasons for Being Interested in These Issues

- Helps us understand the determination of the long-run or normal real interest rate.
- Helps us understand the determination of capital income.
- The investment demand function is important to understanding short-run macroeconomic fluctuations.

II. THE RENTAL MARKET FOR CAPITAL

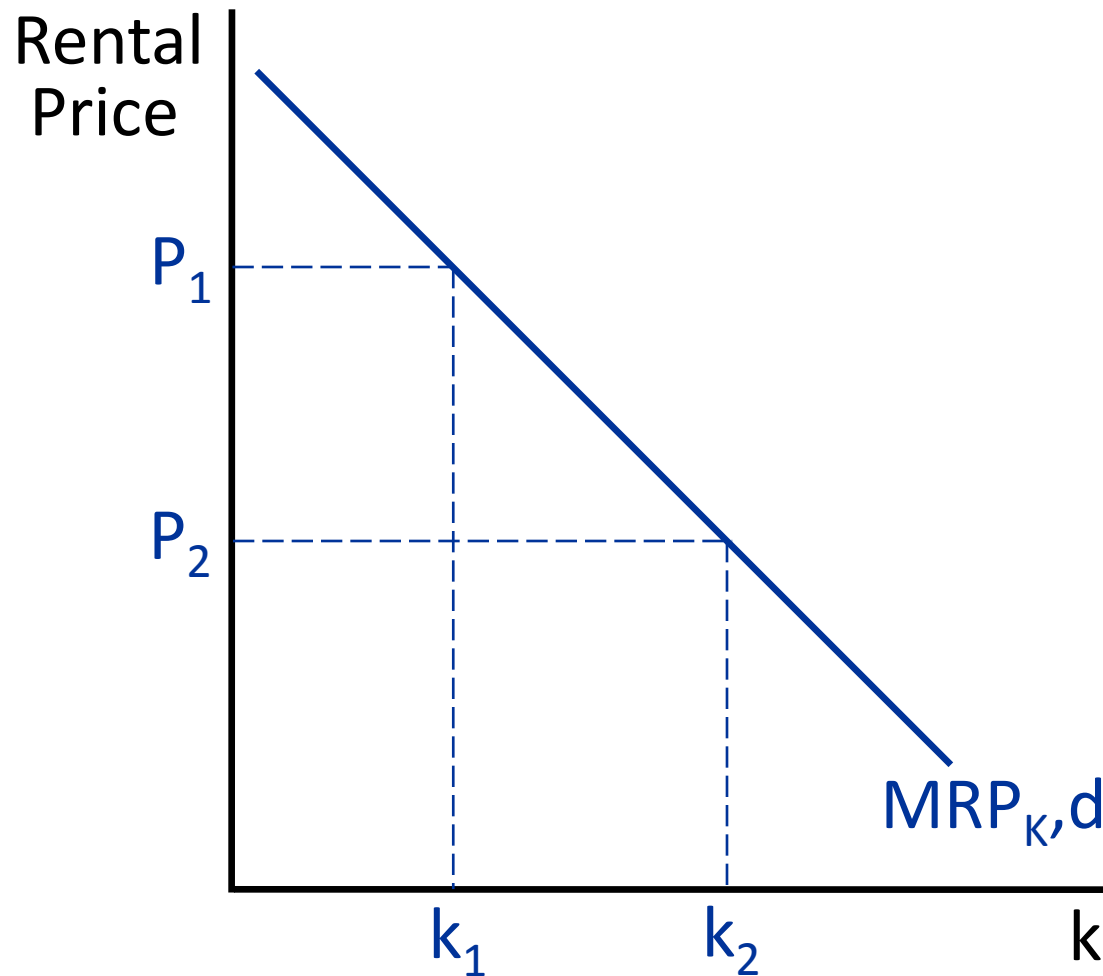
How much capital does a firm want to rent?

- Its decision will be based on profit maximization.
- The firm looks at the MRP of another machine:

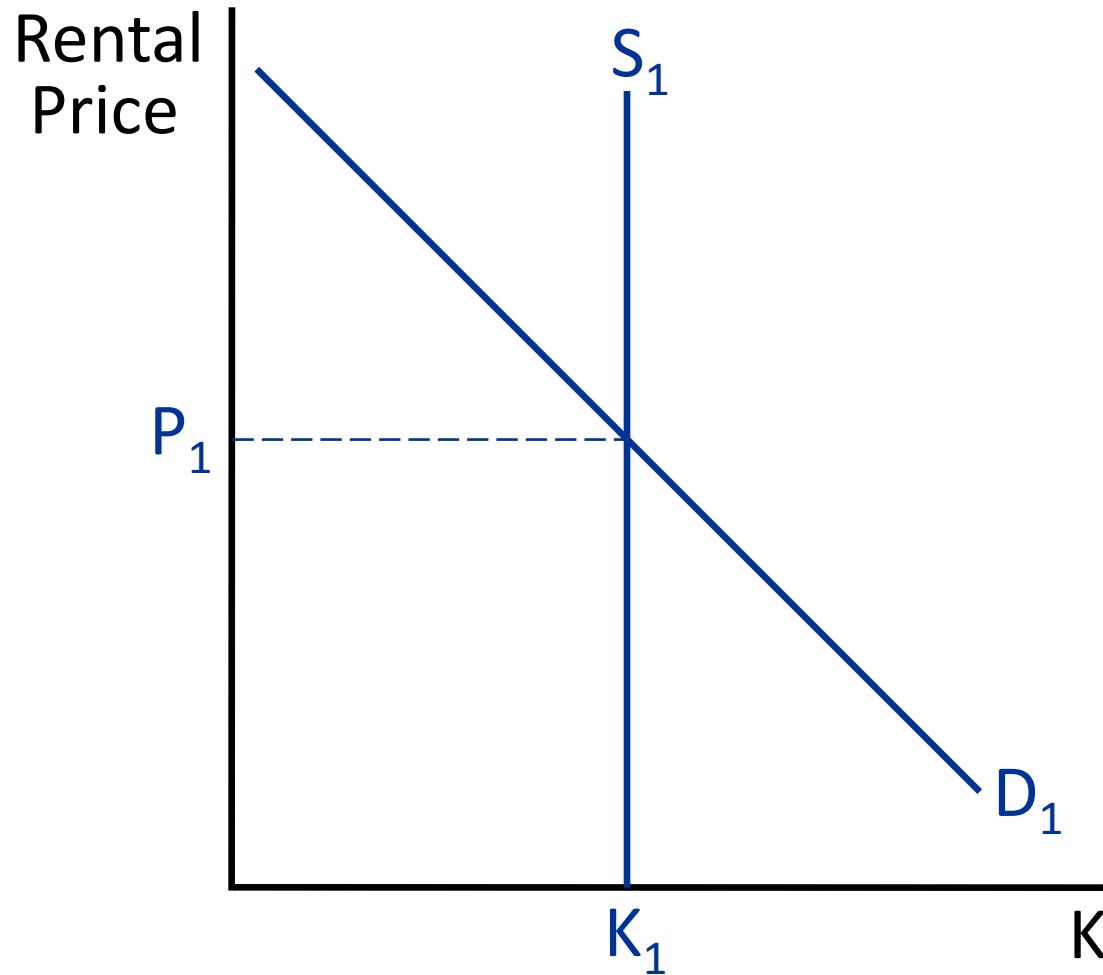
$$MRP_K = MP_K \cdot MR$$

- MRP_K declines as more machines are rented.
- The firm wants to rent machines up to the point where $MRP_K = \text{Rental Price}$.

A Firm's Demand Curve for Rental Capital



Rental Market for Capital



Two Limitations of This Analysis

- It doesn't help us understand how many new machines are purchased—that is, investment.
- It ignores the fact that firms typically buy machines than rent them.

III. PRESENT VALUE

Present Value

- What something to be received in the future is worth today.
- Note: ***To start with, let's assume that there is no inflation or deflation,*** so that the amount of goods and services that can be purchased with a dollar is the same in the future as it is today.

Present Value of a Single Payment to Be Received in the Future

- In general: The present value is how much you would need to put in the bank to get the amount of that payment in the future.

Example: \$1000 to be received a year from now, assuming the interest rate is 3% per year

- The present value, x , is the solution to:

$$x(1 + .03) = \$1000$$

$$x = \frac{\$1000}{(1 + .03)}$$

$$x = \$971$$

Example: Present value of \$1000 one year from now, assuming the interest rate is 8% per year

$$x(1 + .08) = \$1000$$

$$x = \frac{\$1000}{(1 + .08)}$$

$$x = \$926$$

Example: Present value of \$1000 two years from now, assuming the interest rate is 3% per year

$$x(1 + .03)(1 + .03) = \$1000$$

$$x = \frac{\$1000}{(1 + .03)^2}$$

$$x = \$943$$

Present value of a single payment in the future

$$PV(F) = \frac{F}{(1 + r)^t}$$

- F = future payment
- r = annual interest rate (expressed as a decimal)
- t = number of years in the future the payment is to be received

Example: Present value of \$1000 each of the next three years, assuming the interest rate is 3% per year

$$\frac{\$1000}{(1 + .03)^1} + \frac{\$1000}{(1 + .03)^2} + \frac{\$1000}{(1 + .03)^3}$$

$$\$970 + \$943 + \$915$$

$$= \$2828$$

Present Value of a Constant Stream of Payments

$$PV(\text{Stream of } F\text{'s}) =$$

$$\frac{F}{(1+r)^1} + \frac{F}{(1+r)^2} + \frac{F}{(1+r)^3} + \dots + \frac{F}{(1+r)^t}$$

- F = future payment in each year
- r = annual interest rate (expressed as a decimal)
- t = number of years in the future the last payment is made

Present Value of a Stream of Payments That's Different in Different Years

$$PV(\text{Stream of } F\text{'s}) =$$

$$\frac{F_1}{(1+r)^1} + \frac{F_2}{(1+r)^2} + \frac{F_3}{(1+r)^3} + \dots + \frac{F_t}{(1+r)^t}$$

- F_n = future payment in year n
- r = annual interest rate (expressed as a decimal)
- t = number of years in the future the last payment is made

IV. PURCHASING CAPITAL AND THE INVESTMENT DEMAND CURVE

What a machine is worth to a firm:

$$PV(\text{Stream of } MRP_K\text{'s}) =$$

$$\frac{MRP_K}{(1+r)^1} + \frac{MRP_K}{(1+r)^2} + \frac{MRP_K}{(1+r)^3} + \dots + \frac{MRP_K}{(1+r)^t}$$

- MRP_K = marginal revenue product of capital in each year
- r = annual interest rate (expressed as a decimal)
- t = lifespan of the machine

Profit Maximization Implies:

- Firms want to purchase capital up to the point where:

$$PV(\text{Stream of } MRP_K\text{'s}) = \text{Purchase Price}$$

- Note: If we want to be precise, since firms don't know exactly what the MRP_K 's will be, it's really what they expect the MRP_K 's to be that enters the condition for profit maximization.

Important Relationship

- We focus on the relationship between purchases of new capital and the interest rate.
- Why?
- We refer to purchases of new capital (additions to the capital stock) as investment.

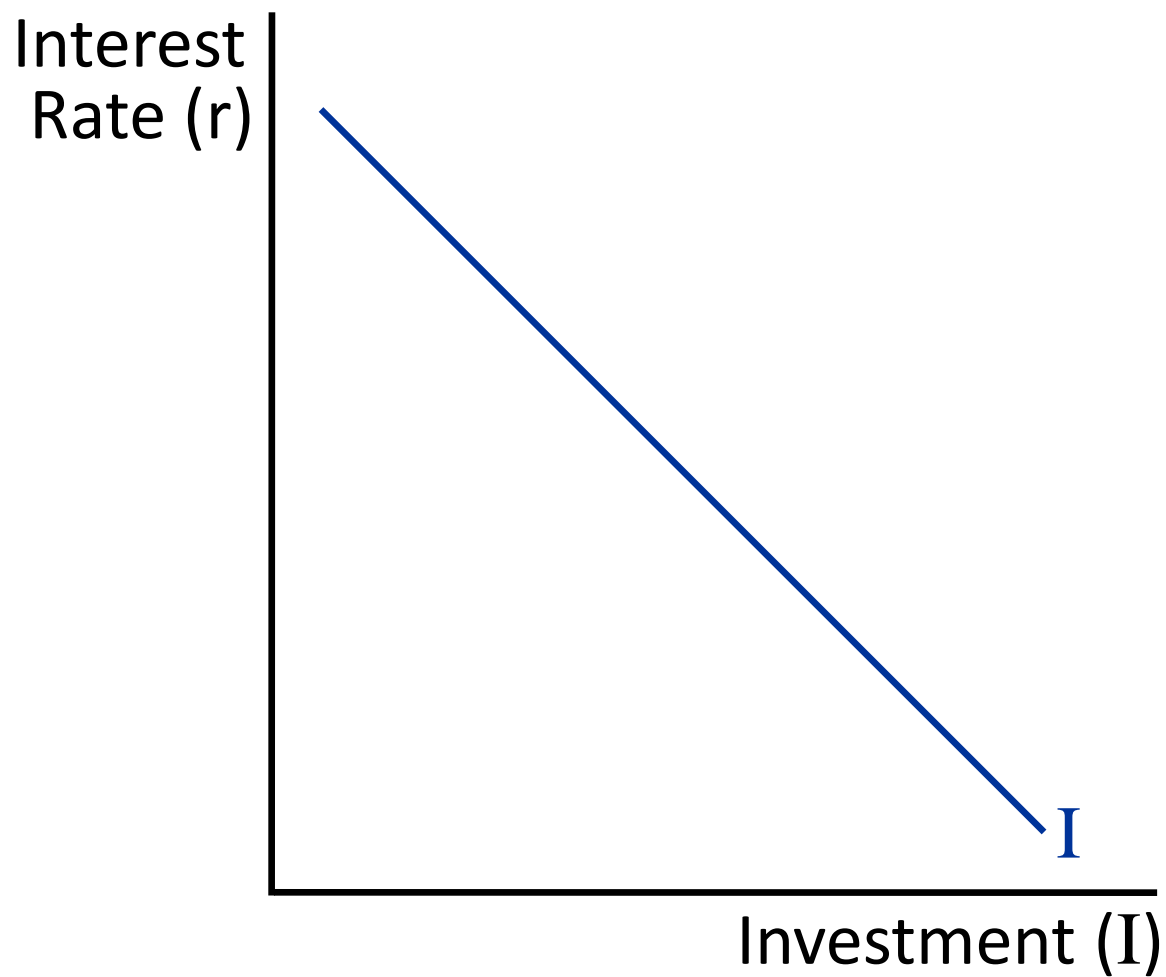
Why is there a negative relationship between purchase of new capital and the interest rate?

- Recall the condition for how much capital a firm wants to buy:

$$PV(\text{Stream of } MRP_K\text{'s}) = \text{Purchase Price}$$

- A term involving r appears in the denominator of expressions for present value: an amount to be received in the future is less valuable when the interest rate is higher.
- An increase in r therefore causes $PV(\text{Stream of } MRP_K\text{'s})$ to fall.
- This makes firms want to buy less capital.

Investment Demand Curve



The Real Interest Rate and the Nominal Interest Rate

- ***Now, let's allow for the possibility of inflation.***
- Recall:
 - “Nominal” means measured in terms of dollars.
 - “Real” means measured in terms of goods and services. (Equivalently, adjusted for changes in prices.)

The Relation between the Real Interest Rate (r) and the Nominal Interest Rate (i)

- The nominal interest rate has two components, compensation for inflation and the real interest rate:

$$i = r + \pi,$$

where π is the inflation rate.

- We can rearrange this as:

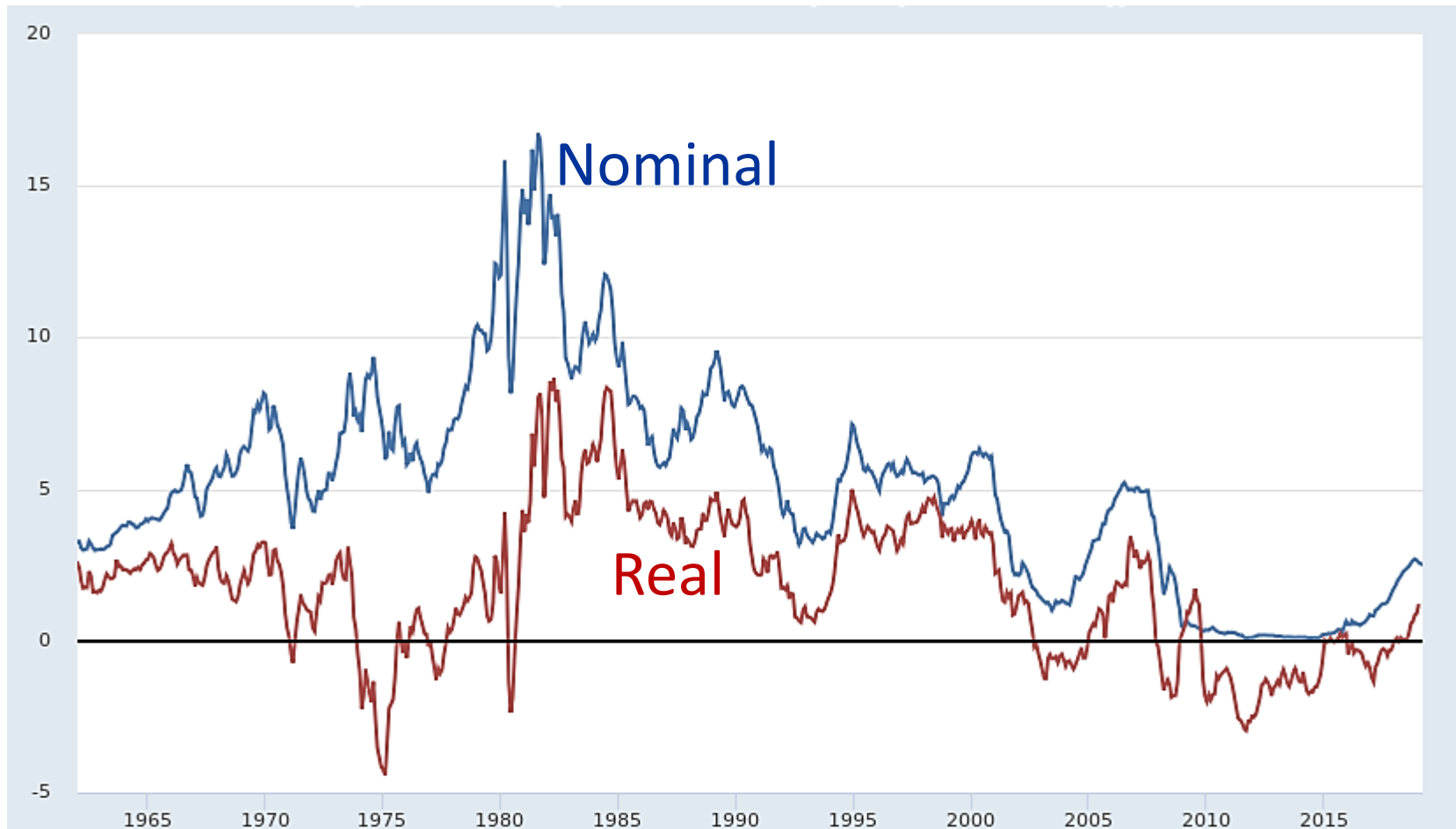
$$r = i - \pi.$$

- Aside: If we want to be precise, the relevant inflation variable is in fact the expected rate of inflation, not the actual rate of inflation.

The Relation between the Real Interest Rate (r) and the Nominal Interest Rate (i)—Example

- Suppose $i = 10\%$ and $\pi = 10\%$.
- Then the nominal interest rate (the percent return you get in dollars) is 10% .
- But the real interest rate (the percent return you get in terms of the purchasing power of what you saved) is 0 .

Nominal and Real Interest Rates (1-year nominal interest rate, and 1-year nominal rate minus 1-year inflation rate)



Source: FRED.

The Real Interest Rate and the Nominal Interest Rate—Computing Present Values of ***Nominal Amounts*** to Be Received in the Future

- To compute the present value of a ***nominal*** amount to be received in the future, you need to use the ***nominal*** interest rate.
- Example: What is the present value of \$10,000 lottery winnings that will be paid a year from now?

$$x(1 + i) = \$10,000$$

$$x = \frac{\$10,000}{(1 + i)}$$

Why Investment Demand Depends on the *Real* Interest Rate—Version 1

- Recall: the firm buys new capital until:

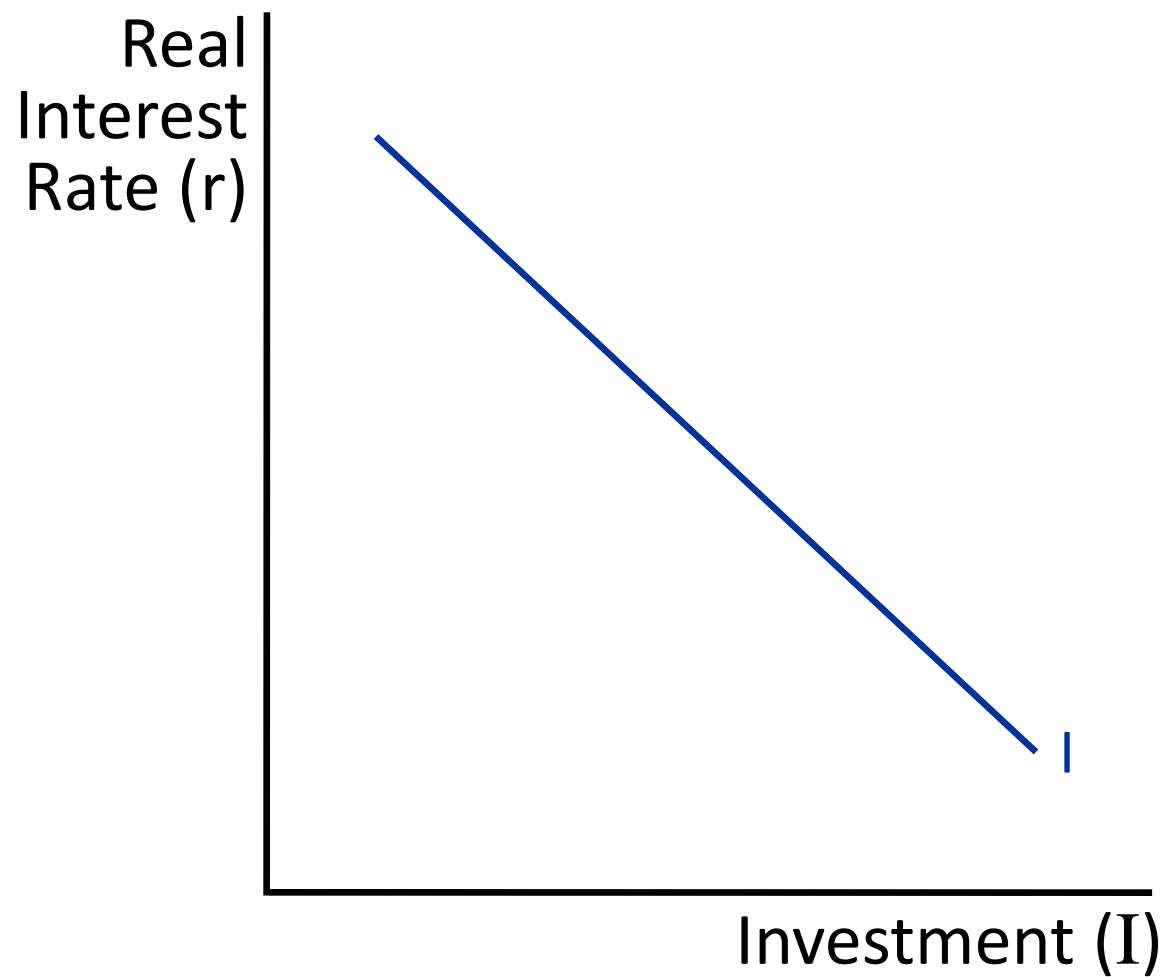
$$PV(\text{Stream of } MRP_K\text{'s}) = \text{Purchase Price}$$

- Think of measuring everything in real (that is, inflation adjusted) terms.
- Then, since we are computing present values of real amounts, the right interest rate to use in computing present values is the real interest rate.
- Thus, if i rises only because π rises, nothing in this expression changes, and so investment demand does not change. So, investment demand depends on the *real* interest rate.

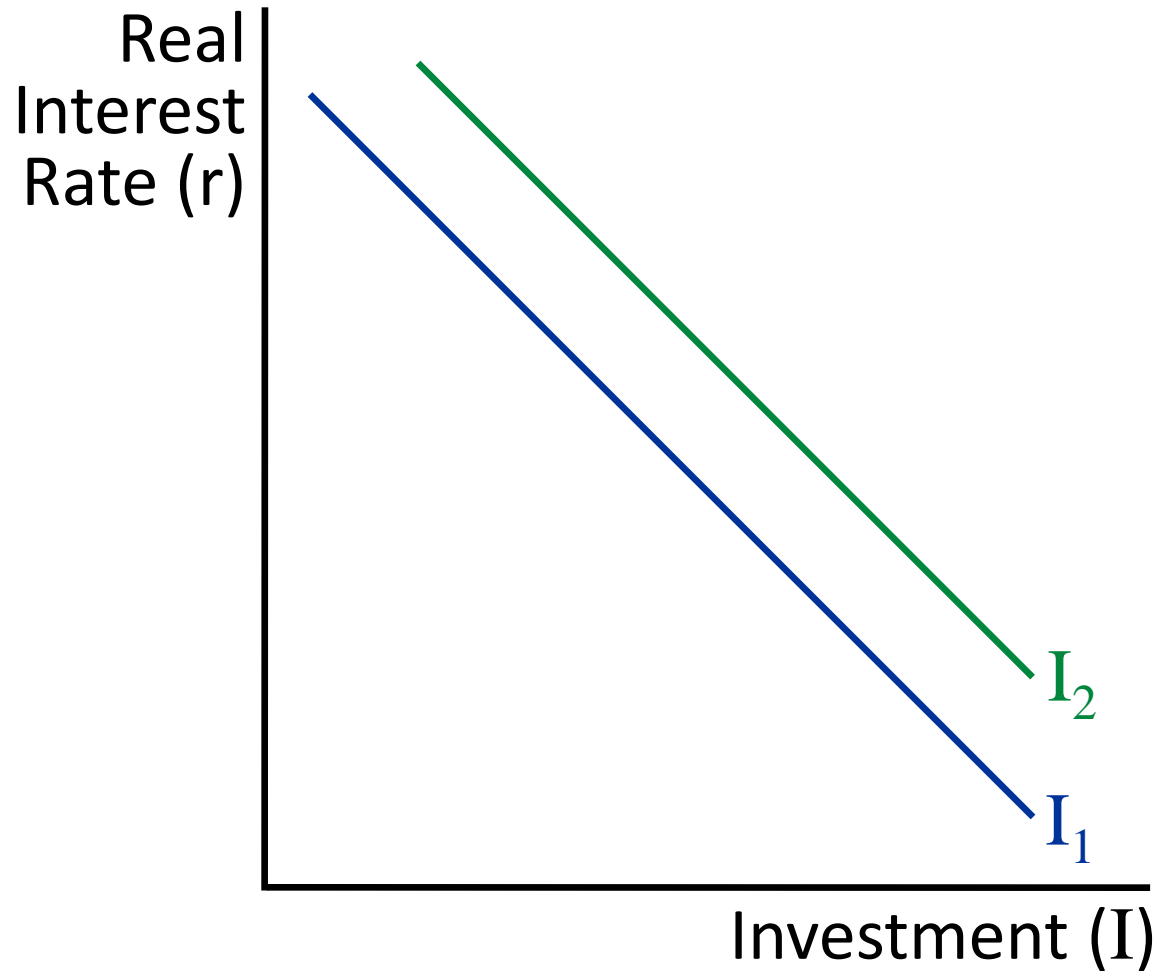
Why Investment Demand Depends on the *Real* Interest Rate—Version 2

- For a competitive firm, $PV(\text{Stream of Future } MRP_K\text{'s})$
$$= \frac{MP_K \bullet P_1}{(1+i)^1} + \frac{MP_K \bullet P_2}{(1+i)^2} + \frac{MP_K \bullet P_3}{(1+i)^3} + \dots + \frac{MP_K \bullet P_t}{(1+i)^t}$$
- Recall that $i = r + \pi$.
- If i rises only because π rises, PV won't change because the P 's will also rise.
- Only if i changes because r changes will PV change.
- Thus: Investment demand depends on the *real* interest rate.

Investment Demand Curve



Shifts in the Investment Demand Curve (Fall in the Purchase Price of Capital)



Shifts in the Investment Demand Curve (Pessimism about Future MRP_K 's)

