UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2018
D. Romer

Problem Set 9
Due in lecture Tuesday, November 13

1. Romer, Problem 8.17.

2. Romer, Problem 8.18.

3. In the firm optimization problem in the *q*-theory model, the transversality condition rules out:

A. Paths where the firm is violating its budget constraint by going further and further into debt.

B. Paths where investment does not satisfy $1 + C'(I(t)) = q(t)$.

C. Paths where the firm is constantly increasing its investment even though the profitability of capital is constantly falling.

D. Paths where investment approaches zero.

4. In the *q*-theory model where the initial value of *K* exceeds its long-run equilibrium value, as the economy moves toward the long-run equilibrium:

A. The $\dot{q} = 0$ locus is shifting to the right and the $\dot{K} = 0$ locus is shifting down.

B. The $\dot{q} = 0$ locus is shifting to the right and the $\dot{K} = 0$ locus is not shifting.

C. The $\dot{K} = 0$ locus is shifting down and the $\dot{q} = 0$ locus is not shifting.

D. None of the above.

5. Consider the q-theory model. Assume that initially the economy is in steady state. Let K^{*OLD} denote the steady-state value of K, and let $\pi^{OLD}(\bullet)$ denote the $\pi(\bullet)$ function.

At some time, which we will call time 0 for simplicity, there is a permanent, unexpected shift of the $\pi(\bullet)$ function. The new function is $\pi^{NEW}(K) = A$ for all K, where $A > \pi^{OLD}(K^{*OLD})$.

a. How, if at all, do the $\dot{q} = 0$ and $\dot{K} = 0$ loci change at t = 0?

b. How, if at all, do q and K change at t = 0?

c. Describe the behavior of q and K after t = 0.

Explain your answers.

EXTRA PROBLEMS (NOT TO BE HANDED IN; COMPLETE ANSWERS MAY NOT BE PROVIDED)

6. Consider an economy that lasts for two periods and that consists of equal numbers of two types of agents, Type A and Type B. The objective function of a representative agent of Type *i* is

$$C_1^i + \beta E\left[C_2^i - \frac{1}{2}a(C_2^i)^2\right], \quad a > 0.$$

where C_t^i is the consumption of an agent of Type *i* in period t. Assume that the C_2^i's are always in the range where marginal utility is positive.

Agents of Type *i* receive an endowment of W_1^i in period 1 and W_2^i in period 2. The W_1^i's are certain and the W_2^i's are uncertain.

Endowments cannot be stored or saved in any way. Thus equilibrium requires $C_1^A + C_1^B = W_1^A + W_1^B$ and $C_2^A + C_2^B = W_2^A + W_2^B$.

a. Suppose the only asset that can be traded is a riskless bond. Specifically, consider an asset that will pay 1 unit for sure in period 2.

i. Set up the problem of an agent of Type *i* choosing how much of the asset to buy. The agent takes P, the price of the asset in period 1 in units of period-1 endowment, as given. The amount bought can be positive or negative (that is, the agent can buy or sell the asset).

ii. Find the demand of an agent of Type *i* for the asset as a function of P and of any relevant parameters (for example, a, β, W_1^i, and the mean and variance of W_2^i).

iii. What is the equilibrium price of the asset? (Hint: What must the sum of the quantities of the asset demanded by the two types of agents be for the market to be in equilibrium?)

b. Suppose agents cannot trade a safe asset, but can trade two risky assets, A and B. The payoff to Asset *i* is W_2^i. Let P_i denote the period-1 price of Asset *i* in units of period-1 endowment. (Thus, if an agent of Type *i* buys Q_A^i of Asset A and Q_B^i of Asset B, his or her consumption is $W_1^i - P_A Q_A^i - P_B Q_B^i$ in period 1, and $W_2^i + Q_A^i W_2^A + Q_B^i W_2^B$ in period 2.)

i. Set up the problem of an agent of Type *i* choosing how much of each of the two assets to buy. The agent takes the prices of the assets in period 1 as given. (As in part (a), the amounts bought can be positive or negative.)

ii. Find the first-order conditions for the problem you set up in part (b)(i).

iii. Assume $W_1^A = W_1^B$, and that W_2^A and W_2^B have the same distribution as one another and are independent. If $P_A = P_B$, will a Type-A agent demand more of Asset A or of Asset B? (A good logical explanation is enough.)

iv. Continue to make the assumptions in part (b)(iii). Get as far as you can in describing the equilibrium quantities ($Q_A^A, Q_B^A, Q_A^B,$ and Q_B^B). (As in part (iii), a good logical argument is enough.)

7. Romer, Problem 8.19.

8. Romer, Problem 8.20.

9. Romer, Problem 9.7.