UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2018
D. Romer

Problem Set 10
Due in lecture Tuesday, November 20

1. Romer, Problem 9.8.

2. (From the 2016 final.) Consider the usual q-theory model. Suppose, however, that firms are taxed on their capital gains (and receive a payment from the government if they have capital losses). Letting τ ($0 < \tau < 1$) denote the tax rate, the flow profits of a firm with quantity of capital $k(t)$ at time t are therefore $\pi\big(K(t)\big)k(t) - I(t) - C\big(I(t)\big) - \tau k(t)\dot{q}(t)$.

a. Explain, either intuitively or mathematically, why the equation of motion for q is $\dot{q}(t) = [rq(t) - \pi\big(K(t)\big)]/(1-\tau)$. (Note: A clear intuitive explanation is enough to get full credit.)

b. Explain, how, if at all, introducing the capital gains tax affects: the $\dot{K} = 0$ locus, the $\dot{q} = 0$ locus, and the saddle path.

c. Suppose there is a tax on capital gains but no payment from the government in the event of capital losses. What is the equation of motion for q in this case?

3. Romer, Problem 9.13, parts (b), (c), and (d).

4. Romer, Problem 10.1.

EXTRA PROBLEMS EXTRA PROBLEMS (NOT TO BE HANDED IN; COMPLETE ANSWERS MAY NOT BE PROVIDED)

5. Romer, Problem 9.14.

6. Romer, Problem 9.15.

7. ***The saddle-path of the q-theory model.*** Consider the two equations of the q-theory model,

$$\dot{q}(t) = rq(t) - \pi\big(K(t)\big), \qquad \dot{K}(t) = f\big(q(t)\big).$$

a. Define the steady state of the model, $(\bar{q}, \bar{K})$. Show that the model's linear (Taylor) approximation in the neighborhood of the steady state takes the form:

$$[\dot{q}, \dot{K}]' \approx \text{G}[q - \bar{q}, K - \bar{K}]',$$

where G =

A	B
C	0

Be sure to show how A, B, and C depend on exogenous parameters, the steady-state values of q and K, and/or the properties of $\pi(\cdot)$ and $f(\cdot)$.

b. Show that the characteristic roots of the preceding 2x2 matrix are:

$$\lambda_1, \lambda_2 = \frac{r \mp \sqrt{r^2 - 4f'(1)\pi'(\bar{K})}}{2},$$

where $\lambda_1 > 0$ and $\lambda_2 < 0$. Please indicate why the second condition holds.

c. Show that the eigenvectors of the matrix G are proportional to the matrix

X =

$\lambda_1/f'(1)$	$\lambda_2/f'(1)$
1	1

d. Define $[\tilde{q}, \tilde{K}]' \equiv \text{X}^{-1}[q - \bar{q}, K - \bar{K}]'$, and note that this implies that $[\dot{\tilde{q}}, \dot{\tilde{K}}]' = \text{X}^{-1}[\dot{q}, \dot{K}]'$. Explain how that change of variables enables us to write the solution of our differential equation system in the form $[\tilde{q}(t), \tilde{K}(t)]' = [\tilde{q}(0)e^{\lambda_1 t}, \tilde{K}(0)e^{\lambda_2 t}]'$ for arbitrary initial conditions $\tilde{q}(0)$ and $\tilde{K}(0)$.

e. From this last relationship deduce that:

$$q(t) - \bar{q} = \tilde{q}(0)(\lambda_1/f'(1))e^{\lambda_1 t} + \tilde{K}(0)(\lambda_2/f'(1))e^{\lambda_2 t},$$

$$K(t) - \bar{K} = \tilde{q}(0)e^{\lambda_1 t} + \tilde{K}(0)e^{\lambda_2 t}.$$

f. Recalling that $\lambda_1 > 0$ and $\lambda_2 < 0$, identify the initial condition that will ensure the economy is on the convergent saddle-path in the usual phase diagram with K on the horizontal axis and q on the vertical axis.

g. For our linear approximation above, express the (linear) equation for the saddle-path in the form

$$q(t) - \bar{q} = \Omega[K(t) - \bar{K}]$$

for an appropriate constant slope $\Omega < 0$. Be sure to show how Ω depends on the model parameters, the steady-state values of q and K, and/or the properties of $\pi(\cdot)$ and $f(\cdot)$.

h. Recall that

$$\lambda_2 = \frac{r - \sqrt{r^2 - 4f'(1)\pi(\bar{K})}}{2}.$$

Discuss which parameters and steady state values affect the slope of the saddle-path. How do they impact the slope? Why?