

Problem Set 8  
Due in lecture Tuesday, November 6

1. Romer, Problem 8.16. (Note: I leave everything about the programming to you. As always, you're welcome to work with others. But any programming must be done collectively, not by delegation or division of labor. Please submit a printout of your code with your problem set. If you use MATLAB, Ben might look at your code and provide feedback if you go astray. If you use other software, he definitely won't be able to. Also, save your code—there will be more on the next problem set!)

2. (From the Fall 2016 final.) Consider a consumer who lives for two periods, 0 and 1. In period 0, the consumer has wealth of  $W$ , which they can allocate between two assets: a safe asset paying a real interest rate of 0 for sure, and a risky asset whose return is normally distributed with mean  $\mu$  and variance  $V_r$ . The consumer's period-1 consumption is their labor income plus their wealth,  $C = Y + S + (1 + r)X$ , where  $Y$  is their labor income,  $S$  is their holdings of the safe asset,  $X$  is their holdings of the risky asset, and  $r$  is the realization of the return on the risky asset.

Importantly,  $Y$  is random and potentially correlated with the return on the risky asset.  $Y$  is normally distributed with mean  $\bar{Y}$  and variance  $V_Y$ , and the covariance between the return on the risky asset and  $Y$  is  $\sigma_{rY}$ . The consumer seeks to maximizes the expected value of period-1 utility, which takes the constant absolute risk aversion form,  $U = -e^{-2\gamma C}$ .

a. Use the budget constraint,  $S + X = W$ , to express expected utility as a function of the consumer's choice of  $X$  and the parameters of the model ( $W$ ,  $\mu$ ,  $\gamma$ , and so on). (A potentially useful fact: If a variable  $a$  is distributed normally with mean  $m$  and variance  $q$ , then  $E[e^a] = e^{m+(q/2)}$ .)

b. What is the derivative of expected utility with respect to  $X$  at  $X = 0$ ? If  $\mu > 0$ , does expected utility necessarily increase if the consumer goes from holding none of the risky asset to holding a small positive amount? Explain why or why not.

c. Find the value of  $X$  that maximizes the consumer's expected utility.

(OVER)

3. (From the Fall 2016 final.) The following is a Bellman equation for an intertemporal consumption problem with uncertainty about labor income:

- A.  $V(A_t) = c_t + \beta A_t + e_t$ .
- B.  $V(A_t) = E_t[\beta(1 + r)u'(A_t + y_t - c_t)]$ .
- C.  $V'(A_t) = E_t[\beta u'(A_t + y_t - c_t)]$ .
- D.  $V(A_t) = \max_{c_t} u(c_t) + \beta E_t[V((A_t - c_t)(1 + r) + y_{t+1})]$ .
- E.  $V(A_t) = \max_{c_t} u(c_t) + \frac{1}{1+r}V((A_t - c_t) + y_{t+1})$ .

4. An individual lives for 3 periods. In period 1, his or her objective function is  $U(C_1) + \beta U(C_2) + \gamma U(C_3)$ . In period 2, his or her objective function is  $U(C_2) + \delta U(C_3)$ . The individual's preferences are not time consistent if:

- A.  $\delta \neq \beta$ .
- B.  $\delta \neq \gamma$ .
- C.  $\delta \neq \beta/\gamma$ .
- D.  $\delta \neq \gamma/\beta$ .

#### EXTRA PROBLEMS (NOT TO BE HANDED IN; COMPLETE ANSWERS MAY NOT BE PROVIDED)

5. (The risk-free rate puzzle.) There is considerable evidence that individuals are quite impatient and quite risk averse. In light of this, consider the standard Euler equation relating consumption in periods  $t$  and  $t+1$  under certainty:  $U'(C_t) = [(1 + r)/(1 + \rho)]U'(C_{t+1})$ . Suppose that  $\rho$  is 5 percent, the coefficient of relative risk aversion is 4, and that the growth rate of consumption is 1.5 percent. What must  $r$  be for consumers to be satisfying their Euler equation?

6. Romer, Problem 8.3.

7. Romer, Problem 8.8.

8. Romer, Problem 8.9.

9. Romer, Problem 8.15.