Notes for Econ202A: Consumption

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Disclaimer: These notes are riddled with inconsistencies, typos and omissions. Use at your own peril. Many thanks to Sergii Meleshchuk for spotting and removing many of them.

very low.

A way to address this critique would be to incorporate directly in the regression a term that controls for the importance of precautionary saving, i.e. for the term $\frac{\theta}{2}V_t\Delta \ln c_{t+1}$ in the regression. This is what Dynan (1993) does by adding proxies for income uncertainty. But it is difficult to obtain such estimates in the first place, and if we try to instrument for the precautionary saving motive, we have to be careful to find instruments for precautionary saving that are independent from the interest rate, not an easy task.

4.2 The Buffer Stock Model

Intuitively, precautionary saving tilts-up consumption profiles and therefore leads to more saving and wealth accumulation. Consider a household that faces income uncertainty. If that household has a high wealth level, then heuristically income uncertainty should not matter much and therefore consumption should not be too different from the certainty equivalent framework (CEQ). We know that in that case, what controls the slope of the expected consumption profile (and therefore of subsequent wealth) is whether βR is smaller of greater than 1.

- If $\beta R > 1$, the household is patient and would like to save. In that case, the precautionary and smoothing motive push in the same direction: eventually, the household will manage to accumulate enough assets to insure against income fluctuations. In fact, if $\beta R > 1$ an infinitely lived household would accumulate an unbounded level of assets.
- If $\beta R = 1$, the argument is a bit more subtle, but the result is the same. Here, the household would like to smooth marginal utility. It will be able to do this by accumulating an unbounded amount of wealth.[15] The upshot is that if $\beta R \geq 1$ the model is not terribly interesting: the household would just accumulate vast amounts of wealth, enough to be indifferent to the impact of income fluctuations on marginal utility. This is neither interesting nor realistic!
- The last case is when $\beta R < 1$. In that case, the household is impatient. A CEQ household would choose to consume more today and run down assets. But by running down assets, it increases the strength of the precautionary saving motive since income fluctuations are more likely to impact marginal utility. So this case present an interesting tension: on the one hand, the household would like to save to smooth fluctuations in marginal utility. On the other hand, it wants to consume now and prefers not to accumulate wealth. The result from this tension is that the household will aim to achieve a certain target level of liquid wealth, but not more. Once households have accumulated this target level of wealth, consumption will tend to track income at high

[15]This result is formally established by Schechtman (1975) and Bewley (1977). See Deaton (1991) for a discussion.

frequency (even in response to predictable income change), thus potentially explaining the excess sensitivity puzzle. It can also explain why consumption tracks income at low frequency (explaining the Carroll-Summers (1991) empirical patterns in figures 14 and 15). This is the buffer-stock model.

Let's flesh the details of that model out. Consider a household with standard preferences:

$$U = \max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

and with a budget constraint:

$$a_{t+1} = R(a_t + \tilde{y}_t - c_t)$$

The household faces a constant interest rate R but a stochastic income stream $\{\tilde{y}_t\}$, where we assume for simplicity that $\tilde{y}_t$ is independently identically distributed every period (i.i.d). We assume that $\beta R < 1$ so that, if there was no uncertainty, the household would prefer to consume now and would run down assets over time, and even borrow against future income.

How much would the household borrow? If $y_{\min} \geq 0$ is the lowest possible realization of income every period, then it is immediate that the household would not be able to run its asset levels below $a_{\min} = -y_{\min}R/(R-1)$.[16] If the household borrowed a larger amount at any point in time, there would be a strictly positive probability that it would not be able to repay. In other words, $a_{\min}$ is the natural borrowing limit faced by the household. It is the present value of the lowest possible income the household would receive from now on. Of course, it is possible that the household faces a stronger liquidity constraint than the natural borrowing limit, if access to credit markets is limited. This is a relevant feature of the world since many people face limited access to credit markets.

In order to fix ideas, we are going to consider an extreme case where the household cannot borrow at all. That is, we impose the restriction that:[17]

$$a_t \geq 0$$

If there was no uncertainty, the solution to the household consumption-saving problem would be quite straightforward: it would run down initial assets a_0, then set consumption equal to income. With uncertainty, this is not going to be optimal for the reasons discussed above: it would leave the household exposed to too much fluctuations in marginal utility.

Therefore, there should be some target level of liquid wealth that the household would like to revert to.

[16] This condition derives from the intertemporal budget constraint and the requirement that consumption remain positive.

[17] This corresponds to the natural borrowing limit if $y_{\min} = 0$.

We can write the income fluctuations problem as (see Deaton (1991)):

$$U = \max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to:

$$\begin{aligned} a_{t+1} &= R(a_t + \tilde{y}_t - c_t) \\ a_t &\geq 0 \\ c_t &\geq 0 \end{aligned}$$

It is useful to express the problem in terms of 'cash on hand' x_t, defined as the amount of liquid resources the household has access to at the beginning of the period:

$$x_t = a_t + y_t$$

The constraints of the problem become:

$$\begin{aligned} x_{t+1} &= R(x_t - c_t) + \tilde{y}_{t+1} \\ 0 &\leq c_t \leq x_t \end{aligned}$$

Let's define $v(x_t)$ the value function of this problem. We can write the associated Bellman equation:

$$\begin{aligned} v(x_t) &= max_{c_t} u(c_t) + \beta E_t[v(x_{t+1})] \\ \text{s.t.} & \\ x_{t+1} &= R(x_t - c_t) + \tilde{y}_{t+1} \\ 0 &\leq c_t \leq x_t \end{aligned}$$

The first order condition associated with this Bellman equation is:

$$u'(c_t) = \beta R E_t[v'(c_{t+1})] + \lambda_t$$

where λ_t is the Lagrange multiplier associated with the constraint $c_t \leq x_t$.[18] The complementary slackness condition is:

$$\lambda_t(x_t - c_t) = 0$$

For the usual envelope reasons, the marginal value of cash on hand satisfies:

$$v'(x_t) = \beta R E_t[v'(x_{t+1})] + \lambda_t = u'(c_t)$$

It follows that:

[18]Technically there is another Lagrange multiplier associated with the constraint $c_t \geq 0$, but this one will never bind as long as the Inada conditions are satisfied, so we ignore it here.

- when the credit constraint does **not** bind, the usual Euler equation holds:

$$u'(c_t) = \beta R E_t[u'(c_{t+1})]$$

- when the credit constraint binds, $\lambda_t > 0$ and $c_t = x_t$ and

$$u'(x_t) > \beta R E_t[u'(c_{t+1})]$$

We can summarize both cases as follows:

$$u'(c_t) = \max \left\langle \beta R E_t[u'(c_{t+1})], u'(x_t) \right\rangle \tag{10}$$

The credit constraint $c_t \leq x_t$ operates in two ways:

1. If the household is constrained at time t, it is forced to consume less than desired.
2. The credit constraint also matters, even in periods where it does not bind directly, because of the likelihood that it will bind in the future. Technically, this is encoded in $E_t[u'(c_{t+1})]$. The curvature of marginal utility leads the household to save more to reduce the likelihood of being constrained in the future.

This model cannot be solved in closed form. Instead, we have to resort to numerical techniques to characterize optimal consumption behavior. Denote $c_t = f(x_t)$ the optimal consumption rule followed by the household. It is not a function of time because the problem is recursive and stationary. We can then rewrite the Euler equation as:

$$\begin{aligned} u'(f(x_t)) &= \max \left\langle \beta R E_t[u'(f(x_{t+1}))], u'(x_t) \right\rangle && (11) \\ x_{t+1} &= R(x_t - f(x_t)) + \tilde{y}_{t+1} && (12) \end{aligned}$$

The problem becomes one of solving for the function $f(.)$. The right hand side of equation (14) defines a functional equation:

$$\begin{aligned} T(f)(x) &= u'^{-1}\left(\max \left\langle \beta R E[u'(f(x_{+1}))], u'(x) \right\rangle\right) \\ x_{+1} &= R(x - f(x)) + \tilde{y}_{+1} \end{aligned}$$

where u'^{-1} is the inverse of the marginal utility (assumed well defined). The optimal consumption rule is then a fixed point of the operator $T(f)$:

$$f(x) = T(f)(x)$$

Not surprisingly, the regularity condition that ensures that this operator has a unique fixed point is $\beta R < 1$, i.e. precisely the requirement that the household is impatient.[19]

[19] Technically, this condition ensures that the operator $T(f)$ is a contraction mapping.

Moreover, this fixed point can be obtained by iteration. Suppose that we have a candidate consumption function $c(x) = f^n(x)$. Then we can construct $f^{n+1}(x)$ as

$$f^{n+1}(x) = T(f^n)(x)$$

i.e. as the solution of:

$$u'(f^{n+1}(x)) = \max \langle \beta R E_t[u'(f^n(x_{+1}))], u'(x) \rangle \quad (13)$$
$$x_{+1} = R(x - f^n(x)) + \tilde{y}_{+1} \quad (14)$$

The sequence $f^n(x)$ converges uniformly to $f(x)$, i.e. $\lim_{n\to\infty} ||f^n(x) - f(x)|| = 0$ where $||.||$ is some Euclidean distance.

This is called Euler equation iteration.[20]

Figure 17 shows the optimal consumption rule for this problem for the case where $y_{\min} > 0$. It has the following properties:

- consumption is a function of x, not y.
- below a certain threshold level x^*, the household prefers to consume all its assets: $c = x$. This is because the current marginal utility of consumption is very high.
- above x^*, the consumption rule is concave, and always below the certainty equivalent consumption
- we can represent expected consumption growth $E_t \Delta \ln c$ as a function of cash on hand x. It is a decreasing function:
  - for low levels of wealth, precautionary saving dominate, cash on hand will increase and consumption is expected to grow.
  - For high levels of cash on hand, consumption grows at rate $\beta R < 1$ so cash on hand decreases.
  - The target level of cash on hand can be defined as that level that remains constant (in expectations), i.e. the level x^{**} such that $E[x_{t+1}|x_t = x^{**}] = x^{**}$. Carroll (2012) shows that that expected consumption growth is below 1 and above βR at the target level of cash-on-handx^{**}.[21]
- Even if c is a function of x, once x is close to its target, c will move together with y: if y is expected to decline, then consumption will decline once x declines (not before): predictable movements in y will translate into movements in c.

Figure 18 reports the dynamics of the buffer stock model, as computed by Carroll (2012).

We can summarize the two models as in table 1:

[20] Another approach, called value function iteration works with the value function $v(x_t)$ that solves the Bellman equation.

[21] For more on this, see Carroll (2012), "Theoretical foundations of buffer stock saving."

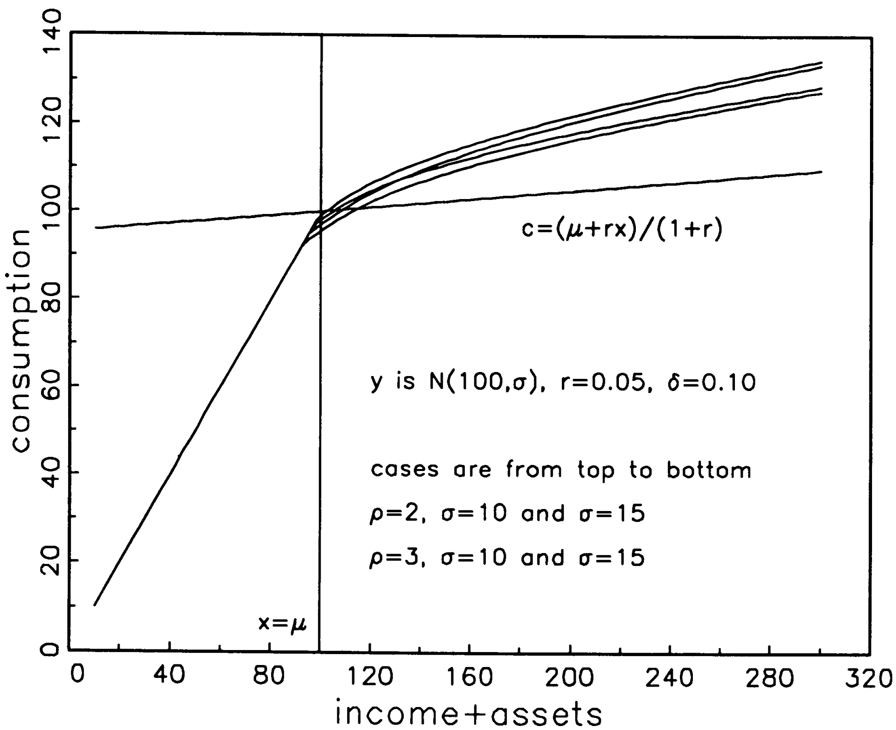

FIGURE 1.—Consumption functions for alternative utility functions and income dispersions.

Figure 17: Deaton (1991), figure 1

4.3 Consumption over the Life Cycle

See Gourinchas & Parker (2002) [GP]. Revisits the question of optimal consumption behavior:

- model with both lifecycle saving motive and precautionary saving motive
- structural estimation of the consumption function, i.e. not relying on Euler equation, or reduced form consumption functions
- estimation based on household level data using income and consumption expenditures

The estimation procedure consists in constructing age-profiles of consumption based on micro data and estimating the parameters of the consumption problems that best replicate these age profiles in the model.

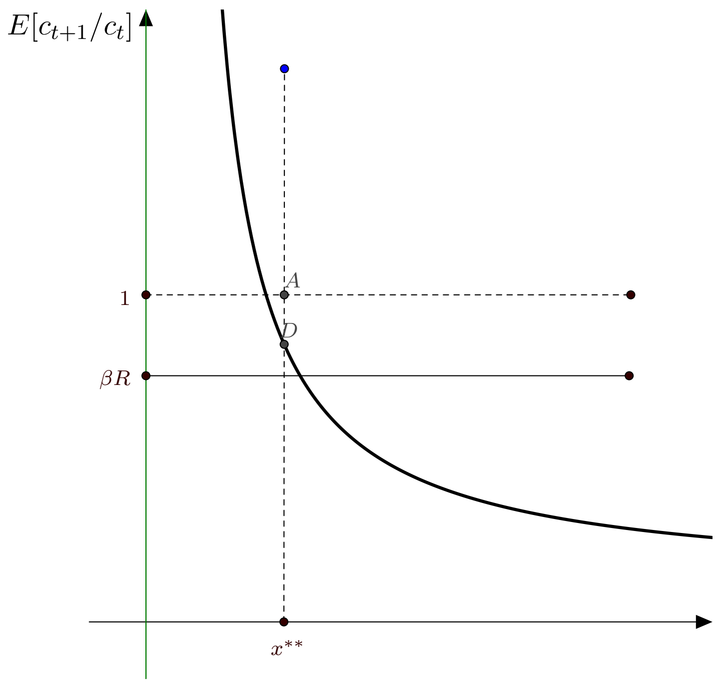

Figure 18: The Buffer Stock Model

Certainty Equivalent	Buffer Stock Model
forward looking	much less forward looking
retirement saving	households will not save for retirement at age 20
consumption and income paths independent	Once you have your buffer, $g_c \approx g_y$
interest rate elasticity	small effect of interest rate
uncertainty does not matter	uncertainty matter

Table 1: Comparing CEQ and Buffer Stock Models

4.3.1 The Model

Each household lives for T periods, works for N periods. GP truncate the problem at retirement by writing:

$$U = E_0[\sum_{t=0}^{N-1} \beta^t u(c_t) + \beta^N V_N(a_N)]$$

subject to:

$$a_{t+1} = R(a_t + y_t - c_t)$$

The function $V_N(.)$ summarizes preferences from retirement onwards, including any bequest motive. GP assume that preferences are CRRA: $u'(c) = c^{-\theta}$. Further, they assume