UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2018
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Problem Set 7
Due at the start of lecture, Thursday, October 25

1. (This is a short answer question from the Fall 2015 midterm.) Suppose I am interested in the impact of European influence (through social infrastructure, culture, technology transfer, and any other channels) on output per worker. I propose to estimate this effect by running (using OLS) the cross-country regression:

$$\ln\frac{Y_i}{L_i} = a + bE_i + e_i,$$

where E_i is the fraction of the population of country i that are native speakers of any European language.

Give <u>one</u> reason that this regression might give a biased estimate of the impact of European influence on output per worker. Explain your reason <u>briefly</u> (no more than a few sentences). In what direction would the problem you identify be likely to bias the estimate, and why?

2. Romer, Problem 4.10.

3. Romer, Problem 4.11.

4. The basic model of consumption under uncertainty (with quadratic utility, and uncertainty only about labor income) implies:

A. The change in income will not be predictable on the basis of past changes in consumption.
B. The change in consumption will not be predictable on the basis of past changes in income.
C. The change in consumption will not be correlated with the current change in income.
D. (A) and (B).
E. (A) and (C).
F. (B) and (C).

5. Romer, Problem 8.5. (However, please skip the second sentence of part d—"Interpret ….")

EXTRA PROBLEMS (NOT TO BE HANDED IN; COMPLETE ANSWERS MAY NOT BE PROVIDED)

6 Romer, Problem 4.12.

7. (Consumption with state-contingent goods.) Consider a consumer whose labor income (which he or she takes as exogenous) is uncertain. Specifically, the consumer's labor income in state s in period t is Y_{st}. The probability that the state in period t is s is π_{st}. Thus, for each t, $\sum_s \pi_{st} = 1$. The realization of the state each period is independent of the realization in all other periods.

The consumer seeks to maximize $E\left[\sum_t \frac{1}{(1+\delta)^t} U(C_t)\right], U'(\cdot) > 0, U''(\cdot) < 0$. The consumer can purchase state-contingent goods and sell his or her state-contingent income. The price of consumption in period t in state s is p_{st}. Thus, we can write the consumer's objective function as $\sum_t \sum_s \pi_{st} \frac{1}{(1+\delta)^t} U(C_{st})$, and his or her budget constraint as $\sum_t \sum_s p_{st} C_{st} \leq \sum_t \sum_s p_{st} Y_{st}$.

a. Set up the consumer's maximization problem, and find the first-order condition for C_{st}.

b. Consider two states in some period *t*, *s*' and *s*''. Under what conditions is consumption the same in the two states? (That is, under what conditions is $C_{s't} = C_{s''t}$?)

c. Consider state *s*' in period *t*' and state *s*'' in period *t*''. Under what conditions is $C_{s't'} = C_{s''t''}$?

d. Consider 2 consumers who differ only in their Y_{st}'s. Show or provide a counterexample to the following claim: If Consumer 1's consumption in one period is greater than Consumer 2's consumption in that period, Consumer 1's consumption in each period is greater than Consumer 2's consumption in the same period.

e. Suppose that both consumers have constant relative risk aversion utility, with the same coefficient of relative risk aversion. What, if anything, can one say about how the ratio of Consumer 1's consumption to Consumer 2's consumption behaves over time?

f. In practice, we often see consumption reversals (that is, one consumer initially having consumption higher than another, but later having lower consumption). List 2 or 3 ways the assumptions of this problem could fail that could make such reversals possible; explain each possibility in no more than a sentence.

g. Suppose that in some period, the realization of *s* is the one that has the highest value of $p_{st}Y_{st}$ for that period for the consumer. How, if at all, will that affect the consumer's consumption in later periods?

8. A consumer facing income uncertainty whose objective function is $E_0[\sum_{t=0}^{\infty} \beta^t U(C_t)]$ and who can borrow and lend at the risk-free interest rate r will satisfy:

A. $U'(C_t) = (1 + r)\beta U'(C_{t+1})$.
B. $U'(C_t) = (1 + r)\beta E_0[U'(C_{t+1})]$.
C. $E_0[U'(C_t)] = (1 + r)\beta E_0[U'(C_{t+1})]$.
D. $U'(C_t) = (1 + r)\beta E_t[U'(C_{t+1})]$.
E. (A) and (C).
F. (B) and (D).
G. (C) and (D).
H. All of the above.

9. Consider a household that will live from 0 to *T* choosing its path of consumption to maximize its lifetime utility, which is given by:

$$\int_{t=0}^{T} e^{-\rho t} u(C(t))dt,$$

where $u(\bullet)$ takes the constant-relative-risk-aversion form:

$$u(C) = \frac{C^{1-\theta}}{1-\theta}, \quad \theta > 0.$$

The household has no initial wealth; its labor income is constant and equal to $\bar{Y}, \bar{Y} > 0$; and the real interest rate is constant and equal to $\bar{r}, \bar{r} > \rho$. As usual, the present discounted value of the household's consumption cannot exceed the present discounted value of the its lifetime resources.

a. What is the present value Hamiltonian?

b. Find the conditions that characterize the solution to the household's maximization problem.

c. Sketch the paths of the household's asset holdings and of $\ln C$ from 0 to *T*. (Note: The problem is not asking you to solve explicitly for asset holdings and $\ln C$ as functions of *t*.)

10. Romer. Problem 2.2.