UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2016
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MIDTERM EXAM

The exam consists of two parts. There are 80 points total. Part I has 12 points and Part II has 68 points.

Some parts of the exam are harder than others. If you get stuck on one part, do the best you can without spending too much time, and then work on other parts of the exam.

On the multiple choice (Part I): If you wish, you may add a <u>BRIEF</u> explanation of your answer to <u>AT MOST ONE</u> question. In that case, your grade on that question will be based on your answer and explanation together. This means that an explanation can either raise or lower a grade.

PART I. Multiple choice (12 points)

In your blue book, give the best answer to 3 of the following 4 questions. (If you answer all 4 questions, your overall score will be based on your average, not on your 3 best scores.)

1. Consider the production function of the Paul Romer model, and suppose that a firm changes from using inputs in equal amounts (for example, $L(i) = b$ for $0 \le i \le A$, where b and A are both positive) to using inputs in unequal amounts with no change in the total quantity of inputs (for example, $L(i) = 2bi/A$ for $0 \le i \le A$). Then the firm's output will:

   A. Increase.
   B. Decrease.
   C. Stay the same.
   D. It is not possible to tell.

2. Among the extensions of their basic model that Gauti Eggertsson and Neil Mehrotra consider in "A Model of Secular Stagnation" are:

   A. The cases of increasing returns to scale production and decreasing returns to scale production.
   B. Both labor income taxation and capital income taxation.
   C. Making the debt limit endogenous both from moral hazard and from adverse selection.
   D. Both a version of the model where agents live for two periods and a version where they live for four periods.
   E. Inequality, both within and across generations.

3. Consider the Diamond overlapping-generations model where k is converging to a balanced-growth-path value from above. Then:

   A. The real interest rate is rising over time.
   B. The real interest rate is falling over time.
   C. The real interest rate is constant over time.
   D. The behavior of the real interest rate is not monotonic.
   E. It is not possible to tell.

4. Hall and Jones's accounting approach to decomposing cross-country income differences into the contributions of physical capital, human capital, and other factors:

   A. Imposes the restriction that, because knowledge is nonrival, if two countries have the same ratio of physical capital to output and the same quantity of human capital per worker, they have the same output per worker.
   B. Finds little evidence of large externalities from physical or human capital.
   C. Takes the capital-output ratio and the quantity of human capital per worker as given.
   D. All of the above.

PART II. Problems (68 points) DO ALL 3 PROBLEMS.

(24 points) 5. Consider the following variant of the Solow model. There are two types of capital, 1 and 2. The production function is $Y(t) = F\big(K_1(t), K_2(t), A(t)L(t)\big)$, with constant returns to scale in the three arguments. Fraction s_i of output is devoted to investment in capital of type i, which depreciates at rate δ_i. Thus, $\dot{K}_1(t) = s_1Y(t) - \delta_1K_1(t)$ and $\dot{K}_2(t) = s_2Y(t) - \delta_2K_2(t)$. The other assumptions are standard: $\dot{L}(t)/L(t) = n$, $\dot{A}(t)/A(t) = g$, and $L(0)$, $A(0)$, $K_1(0)$, and $K_2(0)$ are all strictly positive. Assume $n + g + \delta_1 > 0$, $n + g + \delta_2 > 0$.

a. Define $y \equiv Y/AL$, $k_1 \equiv K_1/AL$, $k_2 \equiv K_2/AL$. Show that one can write y as a function of k_1 and k_2 (analogous to the fact the assumptions of the conventional Solow model allow us to write $y = f(k)$).

b. Derive expressions for $\dot{k_1}(t)$ and $\dot{k_2}(t)$ analogous to the Solow equation, $\dot{k}(t) = sf\big(k(t)\big) - (n + g + \delta)k(t)$. (That is, derive expressions for $\dot{k_1}(t)$ and $\dot{k_2}(t)$ in terms of $k_1(t)$, $k_2(t)$, the exogenous parameters, and the function you derived in part (a).)

c. Now assume that the production function is Cobb-Douglas:

$$Y(t) = K_1(t)^{\alpha}K_2(t)^{\beta}[A(t)L(t)]^{1-\alpha-\beta}, \quad \alpha > 0, \beta > 0, \alpha + \beta < 1.$$

In (k_1,k_2) space, sketch the set of points such that $\dot{k_1} = 0$, and explain your reasoning. Sketch the set of points such that $\dot{k_2} = 0$.

d. Discuss the dynamics of k_1 and k_2 implied by your analysis in (c). (For example, is there a unique (k_1^*, k_2^*) that the economy converges to regardless of its initial situation? Is there a "saddle path" in the model? Are there cases where k_1 and/or k_2 grow without bound? Etc.)

(18 points) 6. Consider a Ramsey-Cass-Koopmans model that is on its balanced growth path.

a. Suppose that at some time, t_0, there is an unexpected, permanent, upward shift of the production function. Concretely, suppose that output is given by $Y(t) = BF\big(K(t), A(t)L(t)\big), B > 0$, and that at t_0 there is an unexpected, permanent increase in B.

i. How, if at all, does this change affect the $\dot{c} = 0$ and $\dot{k} = 0$ loci? Explain.

ii. How, if at all, does the change affect the paths of c and k over time? Explain.

b. Suppose that instead, at time t_0 there is news that starting at time $t_1 > t_0$, there will be a permanent increase in B. How, if at all, does this affect the paths of c and k over time? Explain.

(26 points) 7. (Household optimization with progressive wealth taxation.) Consider a finitely-lived household choosing its path of consumption to maximize lifetime utility. Its objective function is:

$$\int_{t=0}^{T} e^{-\rho t} u(C(t))dt, \quad u'(\bullet) > 0, \quad u"(\bullet) < 0.$$

The household has initial wealth of $A(0)$. The path of its labor income is given by $w(t)$ and the path of the real interest rate is given by $r(t)$. The household takes all of these as given. The household can borrow and lend, but it cannot die in debt. That is, its budget constraint is $A(T) \geq 0$.

Assume, however, that there is a policy of progressive wealth taxation: the household faces a tax on its wealth at rate τ per unit time, with the tax rate a function of its wealth: the tax rate the household faces at time t is given by $\tau(A(t))$, $\tau'(\bullet) > 0$.

a. Explain why the equation of motion for the household's wealth (A) is:

$$\dot{A}(t) = \left[r(t) - \tau\left(A(t)\right)\right]A(t) + w(t) - C(t).$$

b. What is the Hamiltonian? (You are welcome to use either the current-value or the present-value Hamiltonian, but please state which you are using.)

c. Find the conditions for optimality.

d. Use your results to find an expression for $\dot{C}(t)/C(t)$.

e. Discuss how your expression for $\dot{C}(t)/C(t)$ differs from the expression we obtain in a model without wealth taxation (that is, a model where $\tau(A(t)) = 0$ for all A), and explain intuitively how the taxation affects the household's optimizing behavior.