

MIDTERM EXAM

The exam consists of two parts. There are 85 points total. Part I has 18 points and Part II has 67 points.

The exam is designed to take 80 minutes, but you have 90.

Some parts of the exam are harder than others. If you get stuck on one part, do the best you can without spending too much time, and then work on other parts of the exam.

## PART I. Multiple choice and short answer (18 points)

### A. Multiple choice (8 points)

In your blue book, give the best answer to 2 of the following 3 questions. Note:

– If you wish, you may add a BRIEF explanation of your answer to AT MOST ONE question. In that case, your grade on that question will be based on your answer and explanation together. This means that an explanation can either raise or lower a grade.

– If you answer all 3 questions, your overall score will be based on your average, not on your 2 best scores.

1. In the Solow model with all of our usual assumptions, except that  $n + g + \delta = 0$ :

- A.  $k$  would converge to its unique balanced growth path value of 0.
- B.  $k$  would converge to some strictly positive, finite value.
- C.  $k$  would grow without bound.
- D. It is not possible to determine the behavior of  $k$ .

2. Of the following possible regression results concerning the elasticity of long-run output with respect to the saving rate, the one that would provide the best evidence that differences in saving rates are not important to cross-country income differences is:

- A. A point estimate of 5, with a standard error of 2.
- B. A point estimate of 0.1, with a standard error of 0.01.
- C. A point estimate of 0.001, with a standard error of 5.
- D. A point estimate of  $-2$ , with a standard error of 5.

3. One way that Hall and Jones link the two parts of their paper (the levels accounting and the estimation of the effects of social infrastructure) is by:

A. Estimating the effects of social infrastructure on each of the three components they measure in the first part ( $\frac{\alpha}{1-\alpha} \ln \frac{K}{Y}$ ,  $\ln h$ , and  $\ln A$ ).

B. Estimating the effects of social infrastructure on  $\ln A$ , using  $\frac{\alpha}{1-\alpha} \ln \frac{K}{Y}$  and  $\ln h$  as control variables.

C. Using the three components they measure in the first part as instruments in a regression of  $\ln \frac{Y}{L}$  on social infrastructure.

D. Using social infrastructure as an instrument to estimate the effects of the three components on  $\ln \frac{Y}{L}$ .

**B. Short answer (10 points)**

**ANSWER THE FOLLOWING QUESTION.**

4. Suppose I am interested in the impact of European influence (through social infrastructure, culture, technology transfer, and any other channels) on output per worker. I propose to estimate this effect by running (using OLS) the cross-country regression:

$$\ln \frac{Y_i}{L_i} = a + bE_i + e_i,$$

where  $E_i$  is the fraction of the population of country  $i$  that are native speakers of any European language.

Give one reason that this regression might give a biased estimate of the impact of European influence on output per worker. Explain your reason briefly (no more than a few sentences). In what direction would the problem you identify be likely to bias the estimate, and why?

## PART II. Problems (67 points)

### DO ALL 3 PROBLEMS.

(15 points) 5. Consider the model of endogenous knowledge accumulation presented in the book and in lecture for the case of  $\theta < 1$ :

$$\begin{aligned} Y(t) &= (1 - a_L)L(t)A(t), \quad 0 < a_L < 1, \\ \dot{A}(t) &= B[a_L L(t)]^\gamma A(t)^\theta, \quad B > 0, \quad \gamma > 0, \quad \theta < 1. \\ \dot{L}(t) &= nL(t). \end{aligned}$$

Assume  $L(0) > 0$ ,  $A(0) > 0$ . As in the usual model,  $a_L$  is exogenous and constant.

In contrast to the baseline version of the model, assume that the rate of population growth is a decreasing function of the fraction of workers who are engaged in R&D:  $n = n(a_L)$ ,  $n'(\bullet) < 0$ ,  $n(\bullet) > 0$ . (The idea is that, for some reason, scientists on average have fewer children than other workers.)

Suppose the economy is on a balanced growth path, and that there is a permanent increase in  $a_L$ . Sketch the resulting path of  $\ln A$  and what that path would have been without the increase in  $a_L$ . Explain your answer.

(22 points) 6. Consider the Ramsey-Cass-Koopmans model where  $k$  at time 0 (which – as always – the model takes as given) is at the golden rule level:  $k(0) = k^{GR}$ .

a. Sketch the paths of  $c$  and  $k$ . Explain your reasoning.

b. Consider the same initial situation (that is,  $k(0) = k^{GR}$ ), but in the version of the model that includes government purchases. Assume that  $G$  is constant and equal  $\bar{G}$ .  $\bar{G}$  is strictly less than  $f(k^{GR}) - (n + g)k^{GR}$  and strictly greater than  $f(k^*) - (n + g)k^*$  (where  $k^*$  is the level of  $k$  on the balanced growth path the economy would have if  $G$  were constant and equal to 0). Sketch the paths of  $c$  and  $k$ . Explain your reasoning.

**(30 points) 7.** (The social planner's problem in the Ramsey model with adjustment costs.) Consider the Ramsey model. For simplicity, there is no growth in the effectiveness of labor; thus  $A$  is constant and normalized to 1. The key change from the usual version of the model is that there are adjustment costs to changing the capital stock. Specifically, the equation of motion for the capital stock is

$$\dot{K}(t) = Y(t) - C(t) - D(t),$$

where  $D(t)$  is adjustment costs. Adjustment costs per worker are a function of investment per worker, with increasing marginal cost. Specifically:

$$D(t) = L(t)g\left(\frac{Y(t) - C(t)}{L(t)}\right), \quad g(0) = 0, \quad g'(\bullet) > 0, \quad g''(\bullet) > 0.$$

The other assumptions of the model are the same as usual:  $\dot{L}(t) = nL(t)$ ,  $Y(t) = F(K(t), L(t))$ ,  $L(0) > 0$ ,  $K(0) > 0$ . In addition, assume  $n > 0$ .

a. Define  $k \equiv K/L$ ,  $c \equiv C/L$ ,  $f(k) \equiv F(k, 1)$ . Find an expression for  $\dot{k}(t)$  in terms of  $k(t)$ ,  $c(t)$ , the functions  $f(\bullet)$  and  $g(\bullet)$ , and parameters of the model.

b. Consider the problem of a social planner choosing the path of  $c$  to maximize the lifetime utility of the representative household, which is given by

$$\int_{t=0}^{\infty} e^{-\beta t} u(c(t)) dt, \quad u'(\bullet) > 0, \quad u''(\bullet) < 0.$$

What is the current value Hamiltonian?

c. Find the conditions for optimality.

(Note: The problem is asking only for you to find the conditions for optimality, not to do any manipulations of those conditions once you have found them. For example, there is no need to try to find an expression for  $\dot{c}/c$ .)

d. Let  $k^o$  and  $c^o$  denote the balanced growth path values of  $k$  and  $c$  (that is, they are the values of  $k$  and  $c$  such that the conditions in part (c) hold with  $\dot{k} = \dot{c} = 0$ ). Let  $k^*$  and  $c^*$  denote the values of  $k$  and  $c$  on the balanced growth path of an otherwise identical model without adjustment costs. Is  $k^o$  greater than, less than, or equal to  $k^*$ , or is it not possible to tell? Explain intuitively why this is the case.