UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2018
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Problem Set 5
Due at the start of class, Thursday, October 4

1. (From an old midterm.) Consider the model of endogenous knowledge accumulation presented in the book and in lecture for the case of $\theta < 1$:

$$Y(t) = (1 - a_L)L(t)A(t), \quad 0 < a_L < 1,$$

$$\dot{A}(t) = B[a_L L(t)]^{\gamma} A(t)^{\theta}, \quad B > 0, \quad \gamma > 0, \quad \theta < 1.$$

$$\dot{L}(t) = nL(t).$$

Assume $L(0) > 0$, $A(0) > 0$. As in the usual model, a_L is exogenous and constant.

In contrast to the baseline version of the model, assume that the rate of population growth is a decreasing function of the fraction of workers who are engaged in R&D: $n = n(a_L)$, $n'(\bullet) < 0$, $n(\bullet) > 0$. (The idea is that, for some reason, scientists on average have fewer children than other workers.)

Suppose the economy is on a balanced growth path, and that there is a permanent increase in a_L. Sketch the resulting path of $\ln A$ and what that path would have been without the increase in a_L. Explain your answer.

2. (Natural resources in a model of knowledge accumulation.) Consider the following variant of the model of knowledge accumulation and growth in Section 3.2 of *Advanced Macroeconomics*. $R(t)$ denotes use of natural resources at time t, and a_R denotes the fraction of those resources that are used in the R&D sector. The rest of the notation is standard.

$$Y(t) = A(t)[(1 - a_L)L(t)]^{\beta}[(1 - a_R)R(t)]^{1-\beta}, \quad 0 < a_L < 1, 0 < a_R < 1, 0 < \beta < 1,$$

$$\dot{L}(t) = nL(t), \quad n > 0,$$

$$\dot{R}(t) = -\mu R(t), \quad \mu > 0,$$

$$\dot{A}(t) = B[a_L L(t)]^{\gamma}[a_R R(t)]^{\varphi} A(t)^{\theta}, \quad B > 0, \gamma > 0, \varphi > 0.$$

Assume $\theta < 1$. $A(0)$, $L(0)$, and $R(0)$ are all strictly positive.

a. Define $g_A(t) \equiv \dot{A}(t)/A(t)$. Derive an expression for $\dot{g_A}(t)$ in terms of $g_A(t)$ and the parameters.

b. Sketch the function you found in part (a). For what values of g_A is $\dot{g_A} = 0$? For what parameter values and/or initial conditions does g_A converge to each of these values?

c. What is the growth rate of output per person on the balanced growth path as a function of the parameter values and/or initial conditions?

3. Romer, Problem 3.8.

4. Romer, Problem 3.9.

EXTRA PROBLEMS (NOT TO BE HANDED IN; COMPLETE ANSWERS MAY NOT BE PROVIDED)

5. Knowledge accumulation may vary in a complicated way over time. This problem asks you to investigate one way that this might occur.

For simplicity, population is constant. Output at time t is given by Y(t) = (1 – a_L)A(t)L, where Y is output, a_L is the fraction of the population that is engaged in producing knowledge, A is knowledge, and L is population.

Knowledge accumulation is given by the function: $\dot{A}(t) = B_1 a_L L A(t)^\theta$ if A < A*, $\dot{A}(t) = B_2 a_L L$ if A > A*, where A*, B_1, and B_2 are positive parameters, and where θ is a parameter that is assumed to be greater than 1. In addition, B_1 and B_2 are assumed to be such that $\dot{A}$ does not change discontinuously when A reaches A*. This requires that $B_1 a_L L A^{*\theta} = B_2 a_L L$, which is equivalent to $B_2 = B_1 A^{*\theta}$.

The initial level of knowledge, A(0), is assumed to be greater than zero and less than A*.

a. Consider the period when A is less than A*.

i. Define $g_A(t) \equiv \dot{A}(t)/A(t)$. What is $g_A(t)$ as a function of B_1, a_L, L, and A(t)?

ii. Find an expression for $\dot{g}_A(t)$ as a function of $g_A(t)$ and θ.

iii. Is $g_A(t)$ rising, falling, or constant over time?

b. Now consider the period when A is greater than or equal to A*.

i. What is $\dot{A}(t)$?

ii. Is $g_A(t)$ rising, falling, or constant over time?

c. Combine your answers to (a) and (b) to:

i. Sketch the path of the growth rate of output, $\dot{Y}(t)/Y(t)$ over time.

ii. Sketch the path of the log of output, ln Y(t), over time.

6. Romer, Problem 3.5.