UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2018
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Problem Set 2
Due in lecture Thursday, September 13

1. In reading and lecture, we linearized the equations of motion for k and y around k^* and y^*. In many contexts, however, it is more helpful to work with loglinearized than with linearized systems. Thus: Linearize the equation of motion for ln k around ln k^*, and simplify the resulting expression as much as possible.

2. Saving rates may be higher at higher levels of income. This problem asks you to investigate the consequences of this possibility for economic growth.

Consider the Solow model without technological progress. For simplicity, assume that A is one, so that y and k are income per worker and capital per worker.

Now suppose that, in contrast to our usual assumptions:

- The saving rate is zero if income per worker is less than some critical level, $f(\tilde{k})$.
- The saving rate is s (where $s > 0$) if income per worker exceeds $f(\tilde{k})$.

Finally, assume that $sf(\tilde{k})$ is greater than $(n + \delta)\tilde{k}$.

a. Describe how, if at all, this change affects our usual diagram for the Solow model – that is, the diagram showing actual investment per worker and break-even investment per worker as functions of capital per worker.

b. Describe the behavior of output per worker over time if:

i. The initial level of capital per worker, $k(0)$, is between 0 and $\tilde{k}$.

ii. The initial level of capital per worker, $k(0)$, is slightly greater than $\tilde{k}$.

2. Romer, Problem 2.3.

3. Consider an infinitely-lived household maximizing the utility function $\int_{t=0}^{\infty} e^{-\rho t} U\big(C(t)\big)dt$ subject to the usual intertemporal budget constraint. Let $r(t)$ denote the real interest rate at t and let $R(t) \equiv \int_{\tau=0}^{t} r(\tau)d\tau$. Then the Euler equation relating consumption at two dates, A and B (B > A) is:

A. $\frac{\dot{C}(A)}{C(A)} = \frac{[r(t)-\rho]}{\theta}$.

B. $\frac{\dot{C}(A)}{C(A)} = \frac{\dot{C}(B)}{C(B)}$.

C. $U'\big(C(A)\big) = \left[\frac{e^{R(B)-R(A)}}{e^{\rho(B-A)}}\right] U'\big(C(B)\big)$.

D. $U'\big(C(A)\big) = \left[\frac{[r(B)-r(A)]}{[\rho(B-A)]}\right] U'\big(C(B)\big)$.

4. Romer, Problem 9.4. (Note: The "9" is not a typo.)

(NOTE: MORE ASSIGNED PROBLEMS ARE ON THE NEXT PAGE)

5. (From an old final exam.) Consider an infinitely-lived household. The household's initial wealth, A(0) is zero; its labor income is constant and equal to $\bar{Y}, \bar{Y} > 0$; and the real interest rate is constant and equal to $\bar{r} > 0$. The household's flow budget constraint is therefore $\dot{A}(t) = \bar{r}A(t) + \bar{Y} - C(t)$, and, as usual, the present discounted value of the household's consumption cannot exceed the present discounted value of the its lifetime resources.

In contrast to our usual model, however, the household obtains utility not only from consumption, but also from holding wealth. Specifically, its objective function is

$$\int_{t=0}^{\infty} e^{-\rho t}[u(C(t)) + v(A(t))]dt,$$

where $u'(\bullet) > 0$, $u''(\bullet) < 0$, $v'(\bullet) > 0$, $v''(\bullet) < 0$, and $\rho > 0$.

a. For this part only, assume $\rho = \bar{r}$. Without doing any math, explain whether $C(0)$ will be less than, equal to, or greater than $\bar{Y}$, or whether it is not possible to tell.

b. What is the current value Hamiltonian?

c. Find the conditions that characterize the solution to the household's maximization problem. Use them to find an expression for $\dot{C}(t)/C(t)$ that does not involve the costate variable.

6. Romer, Problem 2.7.

7. In the Ramsey-Cass-Koopmans model, the one-time destruction of half of the economy's capital stock:

A. Shifts the $\dot{c} = 0$ locus to the left and does not affect the $\dot{k} = 0$ locus.
B. Shifts the $\dot{k} = 0$ locus down and does not affect the $\dot{c} = 0$ locus.
C. Shifts the $\dot{c} = 0$ locus to the left and shifts the $\dot{k} = 0$ locus down.
D. Does not affect either the $\dot{c} = 0$ locus or the $\dot{k} = 0$ locus.

EXTRA PROBLEMS (NOT TO BE HANDED IN; COMPLETE ANSWERS MAY NOT BE PROVIDED)

8. Show that on the balanced growth path of the Solow model, $K/Y = s/(n + g + \delta)$.

9. Consider an economy described by the Solow model that is on its balanced growth path. Assume that the saving rate is s_0. Now suppose that from time t_0 to time t_1, the saving rate rises gradually from s_0 to s_1 (where $s_1 > s_0$), and then remains at s_1.

Sketch the resulting path over time of log output per worker. For comparison, also sketch on the same graph: (i) the path that log output per worker would have followed if the saving rate had remained at s_0; (ii) the path that log output per worker would have followed if the saving rate had jumped discontinuously from s_0 to s_1 at time t_0 (and remained at s_1).

Explain your answer.

10. Romer, Problem 1.12.

11. Romer, Problem 2.2.

12. Romer, Problem 2.4.

13. (Note: This problem is challenging.) Romer, Problem 1.14.