

Problem Set 1

Due at the start of lecture, Tuesday, September 4

Problem set ground rules:

- You may work with classmates, but: (1) You must try the problems on your own first; (2) You must write up your answers yourself, in your own words.; (3) If you work with others, you should thank them in an acknowledgment footnote at the start of your problem set.

- Looking at already written answers to the problems in any form is not permitted and would be considered academic misconduct.

1. (This is from the 2016 midterm.) Consider the following variant of the Solow model. There are two types of capital, 1 and 2. The production function is $Y(t) = F(K_1(t), K_2(t), A(t)L(t))$, with constant returns to scale in the three arguments. Fraction s_i of output is devoted to investment in capital of type i , which depreciates at rate δ_i . Thus, $\dot{K}_1(t) = s_1Y(t) - \delta_1K_1(t)$ and $\dot{K}_2(t) = s_2Y(t) - \delta_2K_2(t)$. The other assumptions are standard: $\dot{L}(t)/L(t) = n$, $\dot{A}(t)/A(t) = g$, and $L(0)$, $A(0)$, $K_1(0)$, and $K_2(0)$ are all strictly positive. Assume $n + g + \delta_1 > 0$, $n + g + \delta_2 > 0$, $s_1 > 0$, $s_2 > 0$, $s_1 + s_2 \leq 1$.

a. Define $y \equiv Y/AL$, $k_1 \equiv K_1/AL$, $k_2 \equiv K_2/AL$. Show that one can write y as a function of k_1 and k_2 (analogous to the fact the assumptions of the conventional Solow model allow us to write $y = f(k)$).

b. Derive expressions for $\dot{k}_1(t)$ and $\dot{k}_2(t)$ analogous to the Solow equation, $\dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t)$. (That is, derive expressions for $\dot{k}_1(t)$ and $\dot{k}_2(t)$ in terms of $k_1(t)$, $k_2(t)$, the exogenous parameters, and the function you derived in part (a).)

c. Now assume that the production function is Cobb-Douglas:

$$Y(t) = K_1(t)^\alpha K_2(t)^\beta [A(t)L(t)]^{1-\alpha-\beta}, \quad \alpha > 0, \beta > 0, \alpha + \beta < 1.$$

In (k_1, k_2) space, sketch the set of points such that $\dot{k}_1 = 0$, and explain your reasoning. Sketch the set of points such that $\dot{k}_2 = 0$.

d. Discuss the dynamics of k_1 and k_2 implied by your analysis in (c). (For example, is there a unique (k_1^*, k_2^*) that the economy converges to regardless of its initial situation? Are there cases where k_1 and/or k_2 grow without bound? Etc.)

2. (This is from the 2015 midterm.) In the Solow model with all of our usual assumptions, except that $n + g + \delta = 0$:

- A. k would converge to its unique balanced growth path value of 0.
- B. k would converge to some strictly positive, finite value.
- C. k would grow without bound.
- D. It is not possible to determine the behavior of k .

3. Romer, Problem 1.9.

4. Romer, Problem 1.10.

EXTRA PROBLEMS (NOT TO BE HANDED IN; COMPLETE ANSWERS MAY NOT BE PROVIDED)

5. Describe how, if at all, each of the following developments affects the break-even and actual investment lines in our basic diagram for the Solow model:

- a. The rate of population growth falls.
- b. The rate of technological progress rises.
- c. The production function is Cobb-Douglas, $F(K,AL) = K^\alpha(AL)^{1-\alpha}$, and capital's share, α , rises.
- d. Workers exert more effort, so that output per unit of effective labor for a given value of capital per unit of effective labor is higher than before.

6. (Hicks meets Solow.) Consider the Solow model with one change: the production function is $Y = AF(K,L)$. All other assumptions of the model are unchanged. (The type of technical change assumed in the Solow model is known as labor-augmenting or Harrod-neutral technical change; the form of technical change assumed in this problem is called Hicks-neutral.)

- a. Show what happens if we try to derive a balanced growth path like the one derived in class.
- b. What can you say in the special case $F(K,L) = K^\alpha L^{1-\alpha}$, $0 < \alpha < 1$?

7. In the Solow model, a discontinuous rise in s causes \dot{k} (that is, dk/dt) to:

- A. Increase discontinuously.
- B. Start rising at a faster rate than before, but not to increase discontinuously.
- C. Increase discontinuously if the initial level of k is below its new balanced-growth-path value; decrease discontinuously if the initial level of k is above its new balanced-growth-path value; and not change if the initial level of k is equal to its new balanced-growth-path value.
- D. Decrease discontinuously, but then rise at a faster rate than before.

8. In the Solow model, the long-run effect of an increase in the rate of population growth (relative to what would have happened without the change in population growth) is to:

- A. Lower income per person.
- B. Have no impact on the growth rate of income per person.
- C. Increase the growth rate of total income.
- D. (A) and (B).
- E. (A) and (C).
- F. (B) and (C).
- G. (A), (B), and (C).

9. Consider an economy described by the Solow model. Assume that initially capital and output per unit of effective labor are *less* than their balanced-growth-path values. Now suppose that in this situation, the saving rate rises permanently.

Sketch the resulting path of the log of output per worker and what that path would have been if the saving rate had not changed. Explain your answer.

10. (Problem 4, cont.) Romer, Problem 1.11.