219B – Spring 2018
Problem set on Behavioral Finance – Due on April 18

Question (Behavioral Finance – Noise Traders)

This Question elaborates on the DeLong, Shleifer, Summers, Waldman (*JPE* 1990) paper. The idea is to consider what happens to asset prices when a share of the traders have irrational expectations about future dividends. These traders in the literature are called noise traders. Consider the set-up of DeLong, Shleifer, Summers, Waldman (*JPE* 1990), which I summarize here. There is a share μ of noise traders, $(1-\mu)$ of arbitrageurs. The arbitrageurs are risk averse and have a short horizon, that is, they have to sell the shares at the end of period to consumer. Formally, consider an OLG model where in period 1 the agents have initial endowment and trade, and in Period 2 they consume. There are two assets with identical dividend r: a safe asset with perfectly elastic supply, whose price we will set to 1 (numeraire), and an unsafe asset in inelastic supply (1 unit) and a price p that is determined by supply and demand. We denote the demand for unsafe asset: λ^a and λ^n. The investors have CARA utility function $U(w) = -e^{-2(\gamma w)}$ with w being the wealth in Period 2, which is what the investor consumes. Compared to the arbitrageurs, the noise traders believe that in period t the asset with have higher return ρ_t.

a) Assume that the wealth w is distributed $N\left(\overline{w}, \sigma_w^2\right)$. Show that maximizing $EU\left(w\right)$ is equivalent to maximizing $\overline{w} - \gamma\sigma_w^2$, that is, the problem reduces to one of mean-variance optimization.

b) Show that arbitrageurs maximize the problem

$$\max(w_t - \lambda_t^a p_t)(1+r) + \lambda_t^a(E_t[p_{t+1}] + r) - \gamma\left(\lambda_t^a\right)^2 Var_t(p_{t+1}).$$

Derive the first order condition and solve for λ_t^{a*}.

c) Show that noise traders maximize the problem

$$\max(w_t - \lambda_t^n p_t)(1+r) + \lambda_t^n(E_t[p_{t+1}] + \rho_t + r) - \gamma\left(\lambda_t^n\right)^2 Var_t(p_{t+1}).$$

Derive the first order condition and solve for λ_t^{n*}.

d) Discuss how the optimal demand of the risky asset will depend on the expected returns $(r + E_t[p_{t+1}] - (1+r)p_t)$, on risk aversion (γ), on the variance of returns $(Var_t(p_{t+1}))$, and on the overestimation ρ_t.

e) Under what conditions noise traders hold more of the risky asset than arbitrageurs do?

f) To solve for the price p_t, we impose the market-clearing condition $\lambda^n\mu + \lambda^a\left(1-\mu\right) = 1$. Use this condition to solve for p_t as a function of $E_t[p_{t+1}]$, $Var_t(p_{t+1})$, and the other parameters.

g) To solve for the equilibrium, assume that the average price is not time-varying (that is, $E_t\left[p_t\right] = E_t\left[p_{t+1}\right] = E[p]$), and take expectations on the right and left of the expression for p_t. Solve for $E[p]$, and substitute into the expression for p_t. Now, use this expression to compute $Var[p_t]$. Finally, substitute the expression for $Var[p_t]$ in the updated expression for p_t. In the end, you should obtain

$$p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{2\gamma\mu^2\sigma_\rho^2}{r(1+r)^2}. \tag{1}$$

h) Analyze how the price p responds to an increase in μ, in ρ_t, in ρ^*, in γ, and in σ_ρ^2. For each of these terms provide intuition.

i) In light of expression (1), comment on the following statement: 'Biases of investors do not matter in financial markets because they do not affect prices'. What are the key assumptions in the set-up driving this result?

j) (Extra credit) The returns for traders of group j ($j = a, n$) are $R^j = (w_t - \lambda_t^n p_t)(1+r) + \lambda_t^n(p_{t+1}+r) - w$. Straightforwardly, this implies that $\Delta R = R^n - R^a = (\lambda_t^n - \lambda_t^a)(p_{t+1} + r - p_t(1+r))$. Solve that $E(\Delta R|\rho_t)$, that is, the expected return to noise traders relative to arbitrageurs conditional on ρ_t, is

$$E(\Delta R|\rho_t) = \rho_t - \frac{(1+r)^2 \rho_t^2}{2\gamma\mu\sigma_\rho^2} \tag{2}$$

k) Using (2), discuss whether it is possible that noise traders outperform in expectations the arbitrageurs, and under what conditions.

l) What is the intuition for why noise traders may outperform in expectations the arbitrageurs?

m) Does your answer in (l) imply that noise traders can achieve a higher expected utility than arbitrageurs? (Note: I intend when utility is evaluated with the actual returns, not with naive expectations that noise traders have)