Consider a subject who is doing a real effort task, and in particular the A-B press task as in DellaVigna and Pope (2017). Assume that her utility function is of the following form:

\[(s + \Delta s + p)e - c(e)\]

where the cost function \(c(e)\) satisfies the following conditions: \(c'(e) > 0, c''(e) > 0\) for all \(e \geq 0\), \(s\) is the baseline intrinsic motivation, \(p \geq 0\) is the piece rate, \(\Delta s\) is the extra incentive due to one of the psychological interventions (such as the receipt of a “gift”).

a) Derive the optimal level of effort. Do you need extra conditions to guarantee that the solution is interior?

b) Consider the power cost function \(c(e) = ke^{\gamma e}(1 + \gamma)\). What is the optimal effort (when interior)?

What is the elasticity of effort with respect to “incentive in total”, i.e. \((s + \Delta s + p)\)?

c) Consider the exponential cost function \(c(e) = k*exp(\gamma e)/\gamma\). What is the optimal effort (when interior)?

What is the elasticity of effort with respect to “incentive in total”, i.e. \((s + \Delta s + p)\)?

d) In light of the results you get from b) and c), comment on the interpretation of \(\gamma\) in different cost functions.

e) Consider (only in this question) the case of altruism vs. warm glow. The utility function is

\[(s + \alpha p_{CH} + a)e - c(e)\]

where \(p_{CH}\) is the piece rate donated to charity for the charity piece rate treatments. What is the interpretation of \(\alpha\) and \(a\)? (Both terms are zero when there is no donation to a charity) Discuss the different response of the two models \((\alpha > 0\) and \(a = 0\), versus \(\alpha = 0\) and \(a > 0\)) to a change in the charity piece rate \(p_{CH}\)?

In addition to DellaVigna and Pope (2017), read DellaVigna (2018) before you answer the following questions:

f) Assume the power cost function. Suppose that in baseline treatment \((\Delta s = 0, p = 0)\), the optimal effort is \(e_0\); in gift exchange treatment \((p = 0\) but \(\Delta s > 0\) to capture the extra motivation due to gift exchange), the optimal effort is \(e_{gift}\). Express \(\frac{e_{gift} - e_0}{e_0}\) as a function of \(k, s, \Delta s, \gamma\). What is the intuition behind this expression? What parameters does one need to know to estimate \(\Delta s\)?

g) Assume the exponential cost function. Suppose that in baseline treatment \((\Delta s = 0, p = 0)\), the optimal effort is \(e_0\); in gift exchange treatment \((p = 0\) but \(\Delta s > 0\) to capture the extra motivation due to gift exchange), the optimal effort is \(e_{gift}\). Express \(e_{gift} - e_0\) as a function of \(k, s, \Delta s, \gamma\). What is the intuition behind this expression? What parameters does one need to know to identify \(\Delta s\)?

h) Some researchers, after imposing some assumptions and parametrization on the utility function, run the following (nonlinear) regression:

\[ln(e_i) = \frac{1}{\gamma}ln(s + \Delta s + p) - \frac{1}{\gamma}X_i\beta + \epsilon_i\]

Where the vector, \(X_i\), is some demographics. Also assume that \(\epsilon_i \perp (s, \Delta s, p, X_i)\) and \(X_i \perp (s, \Delta s, p)\). What is the assumption made about the shape of cost function \(c(.\) to derive this expression from a utility maximization? What about \(k\)? In what sense are subjects in this specification homogeneous? In what sense
are subjects in this specification heterogeneous? (That is, what form of heterogeneity does this specification allow for?)

i) Suppose we get estimates \( \hat{s}, \Delta \hat{s}, \beta, \gamma \) and the assumptions in h) are correct. In light of your result in f), g) and h), can you use these estimates to approximate \( E[\ln(e_{gift}) - \ln(e_0)] \) in case of power cost function and \( E[e_{gift} - e_0] \) in case of exponential cost function?

j) Consider the library task in Gneezy and List (2006), denote the number of books logged by \( e \), baseline intrinsic motivation due to normal wages by \( s \), the extra incentive due to a gift of higher wages by \( \Delta s_{gift} \). Assume people recruited in their task follow the utility function (with power cost function) in this question. Can you identify \( \hat{s} \) and \( \Delta \hat{s} \) using the design in their paper? If not, what extra treatments are needed?
**Question #2**

Now let’s use the data in DellaVigna and Pope (2017) to estimate the model in the first question. For simplicity, assume in this question that subjects in this experiment will be awarded for every single button press. For example, if the treatment is “As a bonus, you will be paid an extra 1 cent for every 100 points that you score”, then you can just assume that subjects behave as if they are awarded 0.01 cent for every single button press. Therefore, you do NOT need to round the number of button press to the nearest one hundred, as the authors did in their code, so you will not get exactly the same answers.

a) Download the data from REStud website. Use MTurkCleanedData.dta (with 9,861 observations in it) to do this exercise.

b) Produce a table that summarizes the average level, and standard deviation of number of button press within each treatment.

c) First consider the estimation of basic parameters using baseline and piece rate treatments assuming the exponential cost of effort function. In this case the utility function is

\[
(s + p)e - \frac{k}{\gamma} \exp(\gamma e - \gamma \epsilon).
\]

Show that this will lead to estimate

\[
e = \frac{1}{\gamma} \ln(s + p) - \frac{1}{\gamma} \ln(k) + \epsilon
\]

Please pick the following three treatments (baseline, 4 cents and 10 cents) to estimate the parameters \(\hat{s}, \hat{k}, \hat{\gamma}\) using non-linear least squares. Explain why you cannot just estimate this with OLS. Compare your estimates to the result in their paper, are the estimates close or comparable?

d) Further use the estimates you get in c) to predict the level of effort in “very low-pay treatment”. Does it work well? How do you interpret this? Is crowd-out present in this context?

e) In light of your result in Question #1 i), use the estimate you get from c) to approximate \(E[e_4 - e_0]\), where \(e_4\) is the effort in 4 cents treatment, \(e_0\) is the effort in baseline treatment. Does the approximation concord with what you observe from summary statistics? Can you explain why the approximation is successful or not?

f) Plug in several other components to the utility function:

\[
(s + \beta \delta t p + \Delta s_{gift} + \Delta s_{ranking} + \Delta s_{tasksigf} + \alpha p_{CH} + a * 0.01)e - \frac{k}{\gamma} \exp(\gamma e - \gamma \epsilon)
\]

Now use the following treatments (baseline, 1 cent, 4 cents and 10 cents, 2 weeks delay, 4 weeks delay, gift exchange, ranking, task significance, Red Cross) to estimate the parameters using NLS. Report your results. (Note: to make \(\alpha\) and \(a\) comparable, rescale the parameter \(a\) as the authors did in their paper)

g) Use the result you get in f) to predict the average level of effort for the following four hypothetical treatments:

“As a bonus, you will be paid an extra 1 cent for every 100 points that you score. After you play, we will show you how well you did relative to other participants who have previously done this task.”

“As a bonus, you will be paid an extra 1 cent for every 100 points that you score. On top of that, you will be paid a bonus of 40 cents regardless of your performance in appreciation to you for performing this task.”

“As a bonus, you will be paid an extra 1 cent for every 100 points that you score. On top of that, the Red Cross charitable fund will be given 10 cent for every 100 points that you score.”

h) Comment on each of the predictions that you made in g). Do you believe the forecast is likely to be accurate? What is your intuition?

i) Redo c) using a minimum distance estimator using as moments the average effort in the baseline, 4 cent and 10 cent treatments. Notice that this estimator is just identified to estimate \(k, s,\) and \(\gamma\). Compare these results to the results you obtained in (c) using NLS.
References

