LECTURE 17 March 15, 2018

CAPITAL AND INTEREST

I. OVERVIEW

- A. The role of capital in growth
- B. Terminology: capital versus investment
- C. Where we are headed

II. RENTAL MARKET FOR CAPITAL

- A. Profit maximization and the demand for rental capital
- B. Supply and equilibrium
- C. Complications when we think about a firm buying rather than renting capital

III. PRESENT VALUE

- A. Time preference and definition of present value
- B. Present value of a single payment to be received in the future
- C. Present value of a stream of payments to be received in the future

IV. PURCHASING CAPITAL AND THE INVESTMENT DEMAND CURVE

- A. Profit maximization and a firm's decision about how many machines to buy
- B. Investment demand curve
- C. The real interest rate and the investment demand curve
 - 1. The distinction between the nominal and real interest rate
 - 2. Why investment demand depends on the real interest rate
- D. Shifts in the investment demand curve

LECTURE 17 Capital and Interest



March 15, 2018

Announcements

Problem Set 4:

- Due at the beginning of lecture next Tuesday (March 20).
- Problem set work session today (March 15),
 4–6 p.m. in 648 Evans.

Midterm 2:

- Thursday, March 22nd, 2:10–3:30.
- You do not need a blue book.

Midterm 2 Logistics:

- If your GSI is Wesley Huang (Sections 111 & 112) or Maxime Sauzet (Sections 103 & 104), go to 10 Evans; if your GSI is Erik Johnson (Sections 113 & 114), go to 101 Life Sciences Addition.
- DSP Students: You should have received an email from the course assistant (Todd Messer) about arrangements. If you haven't, please contact him (messertodd@berkeley.edu).
- Everyone else come to usual room (2050 VLSB).

- Midterm 2 Format: Similar to Midterm 1.
- Midterm 2 Coverage:
 - Everything up to and including lecture on Tuesday, March 20 (Saving and Investment in the Long Run).
 - Lecture, section, textbook, and additional readings.
 - There will be no questions solely about material from before Midterm 1.

- Hints for Studying:
 - Start now!
 - Review lecture notes and slides; study problem set suggested answers.
 - Pose yourself problems.
 - Do the sample midterm by yourself.

- Places to Get Help:
 - Professor and GSI office hours.
 - Note: Professor office hours next week will be *Tuesday, March 20*, 4:00–6:00 p.m.
 - Review session: Tomorrow (Friday, March 16), 5–7 p.m. in the usual lecture room (2050 VLSB).

LECTURE 17 Capital and Interest



I. OVERVIEW

Aggregate Production Function

$$\frac{Y^*}{POP} = \frac{Y^*}{N^*} \cdot \frac{N^*}{POP}$$

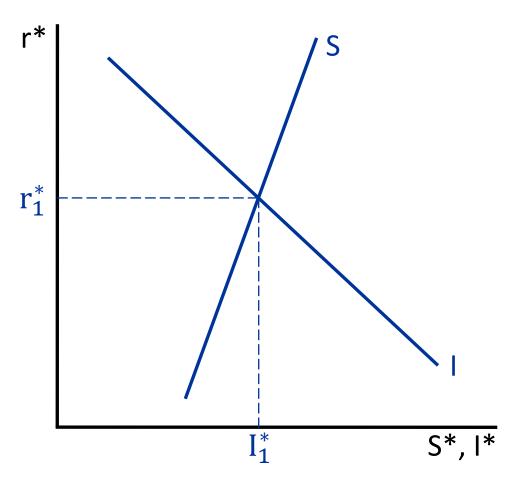
(2)
$$\frac{Y^*}{N^*} = f\left(\frac{K^*}{N^*}, T\right)$$

(3)
$$\frac{Y^*}{POP} = f\left(\frac{K^*}{N^*}, T\right) \cdot \frac{N^*}{POP}$$

Capital and Investment

- Capital: The accumulated stock of aids to the production process that were created in the past.
- Investment:
 - Changes in the capital stock.
 - That is, the construction or purchases of new machines and structures.

Where We're Headed: The Long-Run Saving and Investment Diagram



Here S is saving, I is investment, and r is the real interest rate (and * denotes a long-run value).

Other Reasons for Being Interested in These Issues

- Helps us understand the determination of the long-run or normal real interest rate.
- Helps us understand the determination of capital income.
- The investment demand function is important to understanding short-run macroeconomic fluctuations.

II. THE RENTAL MARKET FOR CAPITAL

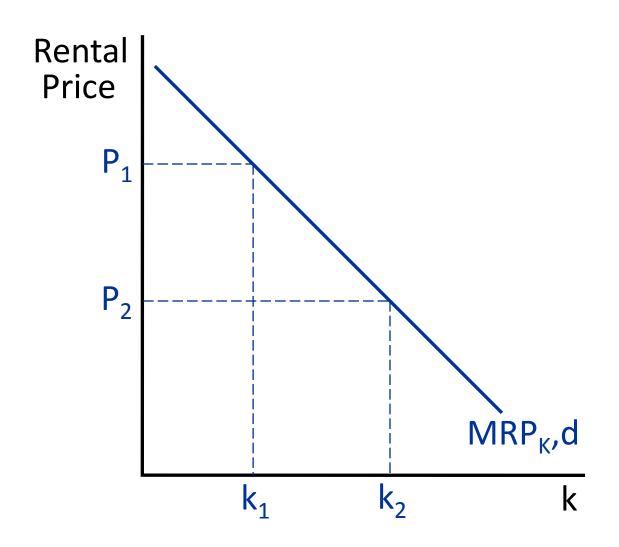
How much capital does a firm want to rent?

- Its decision will be based on profit maximization.
- The firm looks at the MRP of another machine:

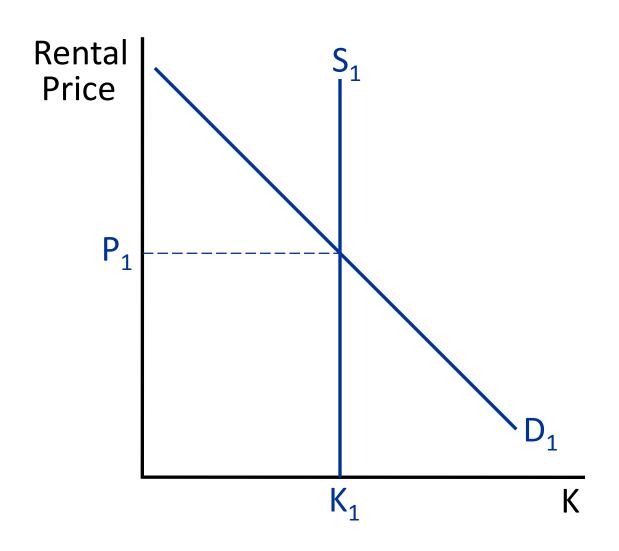
$$MRP_K = MP_K \cdot MR$$

- MRP_K declines as more machines are rented.
- The firm wants to rent machines up to the point where MRP_{κ} = Rental Price.

A Firm's Demand Curve for Rental Capital



Rental Market for Capital



Two Limitations of This Analysis

- It doesn't help us understand how many new machines are purchased—that is, investment.
- It ignores the fact that firms typically buy machines than rent them.

III. PRESENT VALUE

Present Value

- What something to be received in the future is worth today.
- Note: To start with, let's assume that there is no inflation or deflation, so that the amount of goods and services that can be purchased with a dollar is the same in the future as it is today.

Present value of \$100 one year from now:

Assuming the interest rate is 3%.

$$x(1+.03) = 100$$

$$x = \frac{100}{(1+.03)}$$

$$x = $97.09$$

Present value of \$100 one year from now:

Assuming the interest rate is 8%.

$$x(1+.08) = 100$$

$$x = \frac{100}{(1+.08)}$$

$$x = $92.59$$

Present value of \$100 two years from now:

Assuming the interest rate is 3%.

$$x(1+.03)(1+.03) = 100$$

$$x = \frac{100}{(1+.03)^2}$$

$$x = $94.26$$

Present value of a single payment in the future:

$$PV(F) = \frac{F}{(1+r)^t}$$

- F = future payment
- r = interest rate (expressed as a decimal)
- t = number of years in the future the payment is to be received

Present value of \$1000 each of the next three years:

Assuming the interest rate is 3%.

$$\frac{1000}{(1+.03)^{1}} + \frac{1000}{(1+.03)^{2}} + \frac{1000}{(1+.03)^{3}}$$

$$970.09 + 942.60 + 915.14$$

$$= $2827.83$$

Present value of a constant stream of payments:

$$\frac{F}{(1+r)^1} + \frac{F}{(1+r)^2} + \frac{F}{(1+r)^3} + ... + \frac{F}{(1+r)^t}$$

- F = future payment in each year
- r = interest rate (expressed as a decimal)
- t = number of years in the future the last payment is made

Present value of a stream of payments that's different in different years:

$$\frac{F_1}{(1+r)^1} + \frac{F_2}{(1+r)^2} + \frac{F_3}{(1+r)^3} + \dots + \frac{F_t}{(1+r)^t}$$

- F_n = future payment in year n
- r = interest rate (expressed as a decimal)
- t = number of years in the future the last payment is made

IV. PURCHASING CAPITAL AND THE INVESTMENT DEMAND CURVE

What a machine is worth to a firm:

$$PV(Stream of MRP_{K}'s) =$$

$$\frac{MRP_{K}}{(1+r)^{1}} + \frac{MRP_{K}}{(1+r)^{2}} + \frac{MRP_{K}}{(1+r)^{3}} + ... + \frac{MRP_{K}}{(1+r)^{t}}$$

- MRP_K = marginal revenue product of capital in each year
- r = interest rate (expressed as a decimal)
- t = lifespan of the machine

Profit Maximization Implies:

 Firms want to purchase capital up to the point where:

 $PV(Stream of MRP_{\kappa}'s) = Purchase Price$

 Note: If we want to be precise, since firms don't know exactly what the MRP_K's will be, it's really what they expect the MRP_K's to be that enters the condition for profit maximization.

Important Relationship

- We focus on the relationship between purchases of new capital and the interest rate.
- Why?
- We refer to purchases of new capital (additions to the capital stock) as investment.

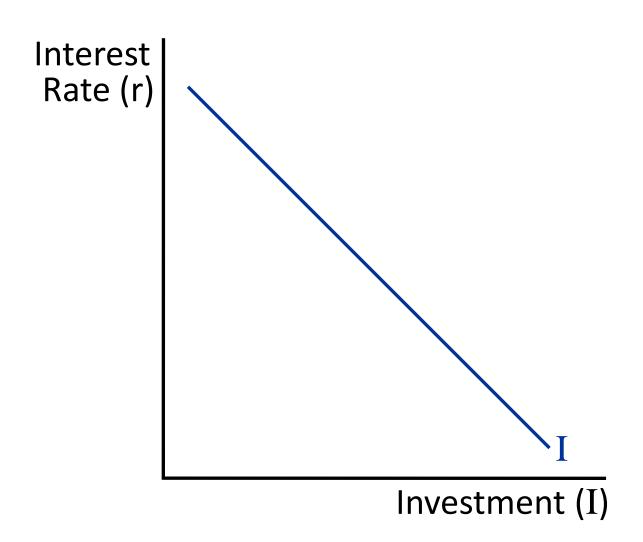
Why is there a negative relationship between purchase of new capital and the interest rate?

 Recall the condition for how much capital a firm wants to buy:

 $PV(Stream of MRP_{\kappa}'s) = Purchase Price$

- A decrease in r causes $PV(Stream of MRP_K's)$ to rise.
- This makes firms want to buy more capital.

Investment Demand Curve



The Real Interest Rate and the Nominal Interest Rate

- Recall:
 - "Nominal" means measured in terms of dollars.
 - "Real" means measured in terms of goods and services. (Equivalently, adjusted for changes in prices.)
- If the nominal interest rate goes up only because inflation goes up, the real interest rate is unchanged.

The Relation between the Real Interest Rate (r) and the Nominal Interest Rate (i)

 The nominal interest rate has two components, compensation for inflation and the real interest rate:

$$i = r + \pi$$
,

where π is the inflation rate.

We can rearrange this as:

$$r = i - \pi$$
.

 Aside: If we want to be precise, the relevant inflation variable is in fact the expected rate of inflation, not the actual rate of inflation.

Nominal and Real Interest Rates (1-year nominal interest rate, and 1-year nominal rate minus 1-year inflation rate)



Source: FRED.

The Real Interest Rate and the Nominal Interest Rate—Computing Present Values of *Dollars* to Be Received in the Future

- To compute the present value of a certain dollar amount to be received in the future, you need to use the nominal interest rate.
- Example: What is the present value of \$10,000 lottery winnings that will be paid a year from now?

$$x(1 + i) = $10,000$$

 $x = \frac{$10,000}{(1 + i)}$

Investment Demand and the Real Interest Rate

- For simplicity, think of a competitive firm, so MR =
 P.
- So: $MRP_K = MP_K \bullet P$.
- If there is inflation, P will be rising over time.

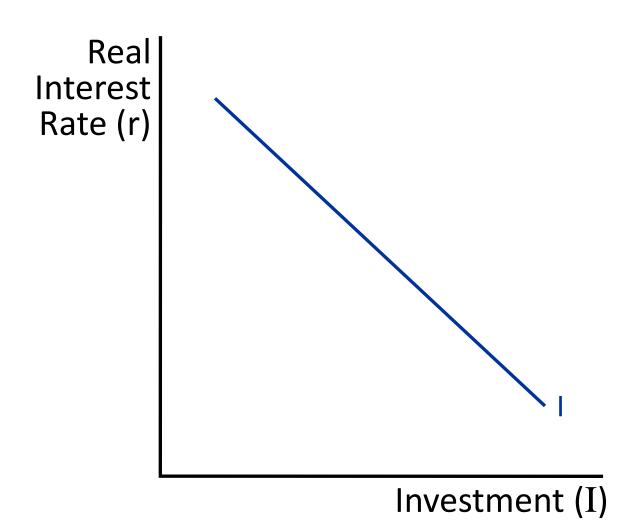
Investment Demand and the Real Interest Rate (continued)

For a competitive firm, PV(Stream of Future MRP_K's)

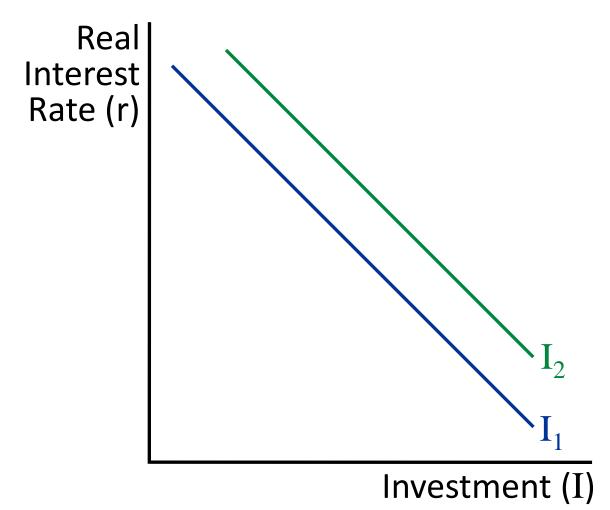
$$= \frac{MP_{K} \cdot P_{1}}{(1+i)^{1}} + \frac{MP_{K} \cdot P_{2}}{(1+i)^{2}} + \frac{MP_{K} \cdot P_{3}}{(1+i)^{3}} + \dots + \frac{MP_{K} \cdot P_{t}}{(1+i)^{t}}$$

- Recall that $i = r + \pi$.
- If i rises only because π rises, PV won't change because the P's will also rise.
- Only if i changes because r changes will PV change.
- Thus: Investment demand depends on the *real* interest rate.

Investment Demand Curve



Shifts in the Investment Demand Curve (Fall in the Purchase Price of Capital)



Shifts in the Investment Demand Curve (Pessimism about Future MRP_K's)

