219B – Problem Set 2 – Reference Dependence

Question #1

In this question and the next we consider the impact of reference dependence on labor supply. Consider Colin, a reference-dependent cab driver deciding how many hours h ($h \geq 0$) per day he intends to work in a single day, with hourly wage w. Colin is risk neutral and thus his daily earnings are $w * h$. Driving more is increasingly costly, so the cost of effort is $\theta h^2/2$. This captures the consumption utility. In addition, Colin has gain-loss utility over the daily earnings, with daily reference point $T > 0$. Colin's utility function therefore is

$$U(h; T, \eta, \lambda, \theta, w) = \begin{cases} wh - \frac{\theta h^2}{2} + \eta (wh - T) & \text{if} \quad wh \geq T \\ wh - \frac{\theta h^2}{2} + \eta\lambda (wh - T) & \text{if} \quad wh < T \end{cases}$$

a) Do a qualitative plot of $U(h; T, \eta, \lambda, \theta, w)$ as a function of the hours h for $\lambda = 2$. If it helps, assume $w = 20$, $T = 100$, $\theta = 1$, and $\eta = 1$. Provide an interpretation for the parameter λ.

b) Do a qualitative plot of $U(h; T, \eta, \lambda, \theta, w)$ as a function of the hours h for $\lambda = 1$. If it helps, still assume $w = 20$, $T = 100$, $\theta = 1$, and $\eta = 1$. This is the utility function of Hank, a cab driver with a standard utility function (though with gain utility).

c) In the next two points, we consider the maximization problem of Hank, the standard cab driver, who wants to determine the optimal number of hours worked h_H^*. Maximize the utility function $U(h; T, \eta, \lambda, \theta, w)$, keeping in mind $\lambda = 1$ for Hank. (Do not make any other assumption on the other parameters) Plot the resulting solution $h_H^* = h_H^*(w|\theta)$ assuming $w = 20$, $T = 100$, $\theta = 1$, and $\eta = 1$ and explain why this is Hank's labor supply function. Does the labor supply curve for Hank depend on η? Comment.

d) Suppose that an econometrician observes repeatedly draws (h_{Ht}, w_{Ht}) from Hank's labor supply and estimates the labor supply function with an OLS regression:

$$h_{Ht} = \alpha + \beta w_{Ht} + \varepsilon_t.$$

What estimates does the econometrician get for α and for β? Provide intuition on the sign and magnitude of the coefficients. Is the model well specified?

e) Now we go back to the case of Colin, with $\lambda = 2$. Maximize the utility function $U(h; T, \eta, \lambda, \theta, w)$, keeping in mind $\lambda = 2$ for Colin. This is harder than for Hank, keep in mind corner solutions (Hint: Distinguish three cases).

f) Do a qualitative plot of the resulting solution $h_C^* = h_C^*(w|T, \theta, \eta, \lambda)$. Assume $w = 20$, $T = 100$, $\theta = 1$, $\eta = 1$, and $\lambda = 2$. Compare Colin's labor supply function with Hank's and comment on the differences.

g) Suppose now that an econometrician observes repeatedly draws (h_{Ct}, w_{Ct}) from Colin's labor supply and estimates the labor supply function with an OLS regression:

$$h_{Ct} = \alpha + \beta w_{Ct} + \varepsilon_t.$$

Is the model well specified? If the econometrician runs the model, what sign of β do you expect to find? Discuss.

h) We are now interested in how the value function of the worker (that is, the utility function at the optimum h^*) varies with the wage w. For Hank (that is, $\lambda = 1$), compute the derivative of the value function with respect to w. (You can use the envelope theorem) What is the sign? Interpret. Now do the same for Colin (that is, $\lambda > 1$), what is the derivative of the value function with respect to w, assuming that the reference point T does not vary with w? What if the reference point moves with w (that is, $T = wh^*$)? Under what model this would make sense?

i) Now assume that in addition to the earnings wh, the workers earn a surprise additional W_0 in income, say from additional tips or unusually long rides. A fraction α of W_0 enters into updating the daily target T. The utility function is now

$$U(h;T,\eta,\lambda,\theta,w,w_0)=\begin{cases} W_0+wh-\frac{\theta h^2}{2}+\eta(wh+W_0-(T+\alpha W_0)) & \text{if} \quad wh+W_0\geq T+\alpha W_0 \\ W_0+wh-\frac{\theta h^2}{2}+\eta\lambda(wh+W_0-(T+\alpha W_0)) & \text{if} \quad wh+W_0< T+\alpha W_0 \end{cases}$$

Consider first the case $\alpha=1$, that is, if the reference point T adjusts one-to-one with the morning surprise. How does the maximization program then compare to the one in point (e)? Plot the resulting solution $h_C^*=h_C^*(w|T,\theta,\eta,\lambda,W_0)$ assuming $w=20$, $T=100$, $\theta=1$, $\eta=1$, and $\lambda=2$, for both the case $W_0=0$ and for the case $W_0=50$. Discuss the intuition.

j) Now consider the case $\alpha=0$, that is if W_0 does not affect the target T. Solve the maximization program. Plot the resulting solution $h_C^*=h_C^*(w|T,\theta,\eta,\lambda,W_0)$ assuming $w=20$, $T=100$, $\theta=1$, $\eta=1$, and $\lambda=2$, for both the case $W_0=0$ and for the case $W_0=50$. Discuss the intuition.

Question #2

a) Camerer et al. (1997) estimates the labor supply of cab drivers using the log-linear specification

$$\log(h_{k,t}) = \alpha + \beta \log(w_{k,t}) + \Gamma X_{k,t} + \varepsilon_{k,t}, \tag{1}$$

where k denotes cab driver k, t denotes day t, and $X_{k,t}$ is a set of controls. What is the approximate K (number of drivers) and T (number of days)? What do they find regarding the elasticity $\hat{\beta}$ of hours worked to wages? What is their conclusion from the finding that $\hat{\beta}$ is negative? Setting aside econometric problems for now, and setting aside the fact that they specify the equation in logs, is their test well-specified? Relate this to the answers you gave in question 1, particularly to questions 1.d and 1.g.

b) We now consider a first econometric issue, called the division bias. Camerer et al. (1997) do not directly observe the wage $w_{k,t}$, but rather they observe $\hat{w}_{k,t} = W_{k,t}/h_{k,t}$, where $W_{k,t}$ is the total earnings for the day. Discuss why this problem can induce a spurious finding of $\hat{\beta} < 0$ in the estimation of (1). How do Camerer et al. (1997) address this problem?

c) A second econometric issue has to do with the estimation of supply and demand. The data on hours worked $h_{k,t}$ and wages $w_{k,t}$ does not just reflect draws from the labor supply curve, but also potentially draws from the labor demand curve. Suppose for example that the different draws across days of $h_{k,t}$ and $w_{k,t}$ reflect purely differences in disutility of effort across days (θ shifts), and assume a downward-sloping labor demand function. Discuss why this problem can induce a spurious finding of $\hat{\beta} < 0$ in the estimation of (1). (A qualitative plot of fixed labor supply and varying demand may help here) Do Camerer et al. (1997) deal with this second issue?

d) Consider now the Farber (2005) paper. What is the approximate K (number of drivers) and T (number of days) in this data set? How does Farber (2005) deal with these two econometric issues? Discuss the estimation strategy and summarize the findings of Farber (2005), focus in particular on Column 4 of Table 5, computing the implied elasticity of stopping with respect to income from the coefficient on (shift income/100). Can he reject the standard model of labor supply? Can he reject the reference-dependent model outlined above?

e) Consider now the Crawford and Meng (2011) paper. What is the approximate K (number of drivers) and T (number of days) in this data set, that is, what data set is it using? Without getting too much in the details, describe the modeling contribution of this paper and the key empirical results.

f) Consider now the Fehr and Goette (2007) paper on bike messengers. Describe briefly the design. How does this paper deal with the two econometric problems outlined above? Summarize the findings of this paper on the number of shifts worked and the effort within a shift.

g) An economist makes the statement "*Fehr and Goette (2007) find that bike messengers work more shifts when paid more. This contradicts the reference dependence story*". Is this right? Discuss in light of your value function computations in question 1.h. Assume that the individual knows the wage w in advance and decides whether to work or not comparing the value function of working to a fixed outside option u.

h) Consider now the Farber (2015) paper. What is the approximate K (number of drivers) and T (number of days) in this data set? Summarize the findings of Farber (2015), focus in particular on Column 1 (day shift) of Table 7, computing the implied elasticity of stopping with respect to income comparing, for example, the stopping for 300-349 compared to the stopping for 200-224 days. Compute an alternative elasticity comparing two other income bins. How do these elasticities compare to the ones implied by Farber (2005)? Are they consistent with reference dependence?

i) Farber (2015) on page 2008 writes "*The evidence is largely consistent with the neoclassical model. Hours are the important driver in the stopping decision, with some evidence of a marginal effect of income (and, hence, income reference points) on the stopping decision on day shifts only.*" Discuss based on your point above.

j) Consider now Thakral and Tô (2017). What data set do they use and thus what is the approximate K (number of drivers) and T (number of days) in this data set? Summarize their findings, in particular in Table 3, Column 1, the impact of income earned in hour 2, 4, 6, and 8.

k) Interpret the findings in Thakral and Tô (2017) relating to your answers in Questions 1.i and 1.j. Discuss under what models of the reference point earnings early in the day (say, in hour 2) may be incorporated into the reference points while earnings later in the day (say, in hour 8) may not. What are the implications for general models of reference point formation?

References

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