Outline

1. Example of General Equilibrium

2. Asymmetric Information: Introduction

3. Hidden Action (Moral Hazard)
1 Example of General Equilibrium

- Consumer 1 has Leontieff preferences:

\[ u(x_1, x_2) = \min(x_1^1, x_2^1) \]

- Bundle demanded by consumer 1:

\[
\begin{align*}
x_1^* &= x_2^* = x^* = \frac{p_1\omega_1^1 + p_2\omega_2^1}{p_1 + p_2} = \\
&= \frac{\omega_1^1 + (p_2/p_1)\omega_2^1}{1 + (p_2/p_1)}
\end{align*}
\]

- Graphically
• Comparative statics:

  – increase in $\omega$

  – increase in $p_2/p_1$:

  \[
  \frac{dx_1^{1*}}{dp_2/p_1} = \frac{\omega_2^1 \left(1 + \left(p_2/p_1\right)\right)}{(1 + \left(p_2/p_1\right))^2} - \frac{\left(\omega_1^1 + \left(p_2/p_1\right)\omega_2^1\right)}{(1 + \left(p_2/p_1\right))^2} = \frac{\omega_2^1 - \omega_1^1}{(1 + \left(p_2/p_1\right))^2}
  \]

  – Effect depends on income effect through endowments:

    * A lot of good 2 $\Rightarrow$ increase in price of good 2 makes richer

    * Little good 2 $\Rightarrow$ increase in price of good 2 makes poorer

• Notice: Only ratio of prices matters (general feature)
• Consumer 2 has Cobb-Douglas preferences:

\[ u(x_1, x_2) = (x_1^2)^{.5} (x_2^2)^{.5} \]

• Demands of consumer 2:

\[ x_1^{2*} = \frac{.5 (p_1 \omega_1^2 + p_2 \omega_2^2)}{p_1} = .5 \left( \frac{\omega_1^2}{p_1} + \frac{p_2 \omega_2^2}{p_1^2} \right) \]

and

\[ x_2^{2*} = \frac{.5 (p_1 \omega_1^2 + p_2 \omega_2^2)}{p_2} = .5 \left( \frac{p_1 \omega_1^2}{p_2} + \omega_2^2 \right) \]
• Comparative statics:

  – increase in $\omega \rightarrow$ Increase in final consumption

  – increase in $p_2/p_1 \rightarrow$ Unambiguous increase in $x_{1}^{2*}$ and decrease in $x_{2}^{2*}$
• Impose Walrasian equilibrium in market 1:

\[ x_1^{1*} + x_2^{2*} = \omega_1^1 + \omega_2^2 \]

This implies

\[
\frac{\omega_1^1 + (p_2/p_1) \omega_2^1}{1 + (p_2/p_1)} + 0.5 \left( \frac{p_2}{p_1} \omega_2^2 \right) = \omega_1^1 + \omega_2^2
\]

or

\[
\frac{0.5 - 0.5 (p_2/p_1)}{1 + (p_2/p_1)} \omega_1^1 + \frac{0.5 (p_2/p_1) + 0.5 (p_2/p_1)^2 - 1}{1 + (p_2/p_1)} \omega_2^1 = 0
\]

or

\[
(\omega_1^1 - 2\omega_2^1) + (\omega_1^1 + \omega_2^1) (p_2/p_1) + \omega_2^1 (p_2/p_1)^2 = 0
\]
• Solution for $p_2/p_1$:

$$\frac{p_2}{p_1} = \frac{-\left(\omega_1^1 - 2\omega_2^1\right) + \sqrt{\frac{\left(\omega_1^1 + \omega_2^1\right)^2}{-4\left(\omega_1^1 - 2\omega_2^1\right)\omega_2^1}}}{2\left(\omega_1^1 - 2\omega_2^1\right)}$$

• Some complicated solution!

• Problem set has solution that is easier to compute (and interpret)
2 Asymmetric Information: Introduction

- Nicholson, Ch. 18, pp. 641-645

- Common economic relationship

- Contract between two parties:
  - Principal
  - Agent

- Two parties have asymmetric information
  - Principal offers a contract to the agent
  - Agent chooses an action
  - Action of agent (or his type) is not observed by principle
• Example 1: *Manager and worker*
  
  – Manager employs worker and offers wage
  
  – Worker exerts effort (not observed)
  
  – Manager pays worker as function of output

• Example 2: *Car Insurance*
  
  – Car insurance company offers insurance contract
  
  – Driver chooses quality of driving (not observed)
  
  – Insurance company pays for accidents

• Example 3: *Shareholders and CEO*
  
  – Shareholders choose compensation for CEO
  
  – CEO puts effort
  
  – CEO paid as function of stock price
• In all of these cases (and many more!), common structure

  – Principal would like to observe effort (of worker, of CEO, of driver)

  – Unfortunately, this is not observable

  – Only a related, noisy proxy is observable: output, accident, success

  – Contract offered by principal is function of this proxy

• This means that occasionally an agent that put a lot of effort but has bad luck is ‘punished’

• Also, agents that shirked may instead be compensated

• These principle-agent problems are called hidden action or moral hazard
• Second category (next lecture): *hidden type* or *adverse selection*

• Example 1: *Manager and worker*
  - Manager employs worker and offers wage
  - Worker can be hard-working or lazy

• Example 2: *Car Insurance*
  - Car insurance company offers insurance contract
  - Drivers ex ante can be careful or careless

• Example 3: *Shareholders and CEO*
  - Shareholders choose compensation for CEO
  - CEO is high-quality or thief
• Problem is similar (action is not observed), but with a twist
  
  – *Hidden action*: principal can convince agent to exert high effort with the appropriate incentives
  
  – *Hidden type*: agent’s behavior is not affected by incentives, but by her type

• Different task for principal:
  
  – *Hidden action*: Principal wants to incentivize agent to work hard
  
  – *Hidden type*: Principal wants to make sure to recruit ‘good’ agent, not ‘bad’ one

• Two look similar, but analysis is different

• Start from *Hidden Action*
3 Hidden Action (Moral Hazard)

- Nicholson, Ch. 18, pp. 645-650

- Example 3: *Shareholders and CEO*
  - Division of ownership and control

- Shareholders (owners of firm):
  - Have capital, but do not have time to run company themselves
  - Want firm run so as to maximize profits

- CEO (manager)
  - Has time and managerial skill
  - Does not have capital to own the firm
• If CEO owns the company (private enterprises), problem is solved \( \Rightarrow \) Infeasible in large companies

• Agent chooses effort \( e \) (unobserved)
  
  – Induces output \( y = e + \varepsilon \), where \( \varepsilon \) is a noise term, with \( E(\varepsilon) = 0 \)
  
  – Example: Despite putting effort, investment project did not succeed

• Principal pays a salary \( w \) to the agent
  
  – Salary is a function of output \( y \): \( w = w(y) \)
  
  – Remember: Salary cannot be function of effort \( e \)
• Principal maximizes expected profits

\[ E[\pi] = E[y - w(y)] = e - E[w(y)] \]

• Agent is risk averse and maximizes

\[ E[U(w(e + \varepsilon))] - c(e) \]

- \( c(e) \) is cost of effort: assume \( c'(e) > 0 \) and \( c''(e) > 0 \) for all \( e \)

- Utility function \( U \) satisfies \( U' > 0 \) and \( U'' < 0 \)

- Notice: Agent is risk-averse, Principal is risk-neutral

• Assume \( U(w) = -e^{-\gamma w} \) and \( \varepsilon \sim N(0, \sigma^2) \)

• Can solve explicitly for \( EU(w) \):

\[ EU(w) = -\frac{1}{\sqrt{2\pi\sigma^2}} \int e^{-\gamma w} e^{-\frac{1}{2}\frac{(w-\mu_w)^2}{\sigma_w^2}} dw = \mu_w - \frac{\gamma}{2} \sigma_w^2 \]

[Take this for granted]
• Expected utility of agent is \( EU (w) = \mu_w - \frac{\gamma}{2} \sigma_w^2 \)

• Note: \( \mu_w \) is average salary and \( \sigma_w^2 \) is variance of salary
  
  – Agent likes high mean salary \( \mu_w \)
  
  – Agent dislikes variance in salary \( \sigma_w^2 \)
  
  – Dislike for variance increases in risk aversion \( \gamma \)

• Assume that contract is linear: \( w = a + by = a + be + b\varepsilon \)
  
  – Compute \( \mu_w = E (w) = E [a + be + b\varepsilon] = a + be + bE [\varepsilon] = a + be \)
  
  – Compute \( \sigma_w^2 = Var [a + be + b\varepsilon] = b^2 \sigma^2 \)

• Rewrite expected utility as
  
  \[
  EU (w) = a + be - \frac{\gamma}{2} b^2 \sigma^2
  \]
• Back to Principal-Agent problem

• Solve problem in three Steps, starting from last stage (backward induction)

  – **Step 1** (*Effort Decision*). Given contract $w(y)$, what effort $e^*$ is agent going to put in?

  – **Step 2.** (*Individual Rationality*) Given contract $w(y)$ and anticipating to put in effort $e^*$, does agent accept the contract?

  – **Step 3.** (*Profit Maximization*) Anticipating that the effort of the agent $e^*$ (and the acceptance of the contract) will depend on the contract, what contract $w(y)$ does principal choose to maximize profits?
• **Step 1.** Solve effort maximization of agent:

\[ \max e + be - \frac{\gamma b^2 \sigma^2}{2} - c(e) \]

• Solution:

\[ c'(e) = b \]

• If assume \( c(e) = ce^2/2 \rightarrow e^* = b/c \)

• Check comparative statics

  – With respect to \( b \rightarrow \) What happens with more pay-for-performance?

  – With respect to \( c \rightarrow \) What happens with higher cost of effort?
• **Step 2.** Agent needs to be willing to work for principal

• *Individual rationality* condition:

\[ EU \left( w(e^*) \right) - c(e^*) \geq 0 \]

• Substitute in the solution for \( e^* \) and obtain

\[ a + be^* - \frac{\gamma b^2 \sigma^2}{2} - c(e^*) \geq 0 \]

• Will be satisfied with equality: \( a^* = -be^* + \frac{\gamma b^2 \sigma^2}{2} + c(e^*) \)
• Step 3: Owner maximizes expected profits

\[
\max_{a,b} E[\pi] = e - E[w(y)] = e - a - be
\]

• Substitute in the two constraints: \( c'(e) = b \) (Step 1) and \( a^* = -be^* + \frac{\gamma}{2}b^2\sigma^2 + c(e^*) \) (Step 2)

• Obtain

\[
E[\pi] = e - \left( -be + \frac{\gamma}{2}b^2\sigma^2 + c(e) \right) - c'(e)e
\]
\[
= e + be - \frac{\gamma}{2}b^2\sigma^2 - c(e^*) - c'(e)e
\]
\[
= e + c'(e)e - \frac{\gamma}{2}(c'(e))^2 \sigma^2 - c(e^*) - c'(e)e
\]
\[
= e - \frac{\gamma}{2}(c'(e))^2 \sigma^2 - c(e^*)
\]

• Profit maximization yields f.o.c.

\[
1 - \gamma c'(e) \sigma^2 c''(e) - c'(e) = 0
\]
and hence

\[ c'(e^*) = \frac{1}{1 + \gamma \sigma^2 c''(e^*)} \]

- Notice: This implies \( c'(e^*) < 1 \)

- Substitute \( c(e) = ce^2/2 \) to get

\[ e^* = \frac{1}{c} \frac{1}{1 + \gamma \sigma^2 c} \]

- Comparative Statics:
  - Higher risk aversion \( \gamma \rightarrow \ldots \)
  - Higher variance of output \( \sigma \rightarrow \ldots \)
  - Higher effort cost \( c \rightarrow \ldots \)
• Also, remember $b^* = c'(e^*) = ce^*$ and hence

$$b^* = ce^* = c \frac{1}{1 + \gamma \sigma^2 c} = \frac{1}{1 + \gamma \sigma^2 c}$$

• Notice $0 < b^* < 1$:

  – Agent gets paid increasing function of output to incentivize
  
  – Does not get paid one-on-one ($b = 1$) because that would pass on too much risk to agent
  
  – (Remember $w^* = a^* + b^* y = a^* + b^* e + b^* \varepsilon$)
  
  – Comparative Statics: what happens to $b^*$ if $\gamma = 0$ or $\sigma = 0$? Interpret
• Consider solution when effort is observable

• This is so-called **first best** since it eliminates the uncertainty involved in connecting pay to performance (as opposed to effort)

  - Principal offers a flat wage \( w = a \) as long as agent works \( e^* \)

  - Agent accepts job if

    \[
    a - c(e^*) \geq 0
    \]

  - Principal wants to pay minimal necessary and hence sets \( a^* = c(e^*) \)

  - Substitute into profit of principal

    \[
    \max_{a,b} E[\pi] = e - E[w(y)] = e - a^* = e - c(e)
    \]

  - Solution for \( e^* \):

    \[
    c'(e^*) = 1 \text{ or } e^*_{FB} = 1/c
    \]
• Compare $e^*$ above and $e^*_{FB}$ in first best

• $\rightarrow$ With observable effort (first best) agent works harder
• Summary of hidden-action solution with risk-averse agent:

• **Risk-incentive trade-off:**
  
  – Agent needs to be incentivized ($b^* > 0$) or will not put in effort $e$
  
  – Cannot give too much incentive ($b^*$ too high) because of risk-aversion
  
  – Trade-off solved if
    
    * Action $e$ observable OR
    
    * No risk aversion ($\gamma = 0$) OR
    
    * No noise in outcome ($\sigma^2 = 0$)
  
  – Otherwise, effort $e^*$ in equilibrium is sub-optimal

• Same trade-off applies to other cases
• Example 2: *Insurance* (Not fully solved)

  - Two states of the world: Loss and No Loss
  
  - Probability of Loss is $\pi(e)$, with $\pi'(e) < 0$

    * Example: Careful driving (Car Insurance)

    * Example: Maintaining your house better (House insurance)

    * Agent chooses quantity of insurance $\alpha$ purchased

  - Agent risk averse: $U(c)$ with $U' > 0$ and $U'' < 0$
- Qualitative solution:
  
  - No hidden action $\rightarrow$ Full insurance: $\alpha^* = L$
  
  - Hidden action $\rightarrow$
    
    * Trade-off risk-incentives $\rightarrow$ Only Partial insurance $0 < \alpha^* < L$
    
    * Need to make agent partially responsible for accident to incentivize
    
    * Do not want to make too responsible because of risk-aversion