Outline

1. Oligopoly: Cournot

2. Oligopoly: Bertrand

3. Second-price Auction

4. Auctions: eBay Evidence

5. Dynamic Games
1 Oligopoly: Cournot

- Nicholson, Ch. 15, pp. 534-540

- Back to oligopoly maximization problem

- Assume 2 firms, cost $c_i (y_i) = cy_i$, $i = 1, 2$

- Firms choose simultaneously quantity $y_i$

- Firm $i$ maximizes:

  $$\max_{y_i} p (y_i + y_{-i}) y_i - cy_i.$$ 

- First order condition with respect to $y_i$:

  $$p'_{y_i} \left( y_i^* + y_{-i}^* \right) y_i^* + p - c = 0, \ i = 1, 2.$$
• Nash equilibrium:

  - \( y_1 \) optimal given \( y_2 \);
  - \( y_2 \) optimal given \( y_1 \).

• Solve equations:

\[
p'_Y (y_1^* + y_2^*) y_1^* + p - c = 0 \quad \text{and} \quad p'_Y (y_2^* + y_1^*) y_2^* + p - c = 0.
\]

• Cournot -> Pricing above marginal cost

• Numerical example -> Problem set 5
2 Oligopoly: Bertrand

- Nicholson, Ch. 15, pp. 533-534

- Cournot oligopoly: firms choose quantities

- Bertrand oligopoly: firms first choose prices, and then produce quantity demanded by market

- Market demand function $Y(p)$

- 2 firms

- Profits:

$$\pi_i(p_i, p_{-i}) = \begin{cases} 
(p_i - c)Y(p_i) & \text{if } p_i < p_{-i} \\
(p_i - c)Y(p_i)/2 & \text{if } p_i = p_{-i} \\
0 & \text{if } p_i > p_{-i}
\end{cases}$$
• First show that \( p_1 = c = p_2 \) is Nash Equilibrium

• Does any firm have a (strict) incentive to deviate?

• Check profits for Firm 1

• Symmetric argument for Firm 2
• Second, show that this equilibrium is unique.

• For each of the next 5 cases at least one firm has a profitable deviation

• Case 1. $p_1 > p_2 > c$

• Case 2. $p_1 = p_2 > c$

• Case 3. $p_1 > c \geq p_2$
• Case 4. $c > p_1 \geq p_2$

• Case 5. $p_1 = c > p_2$

• Only Case 6 remains: $p_1 = c = p_2$, which is Nash Equilibrium

• It is unique!
• Notice:

• To show that something is an equilibrium \(\rightarrow\) Show that there is *no* profitable deviation

• To show that something is *not* an equilibrium \(\rightarrow\) Show that there is *one* profitable deviation
• Surprising result of Bertrand Competition

• Marginal cost pricing

• Two firms are enough to guarantee perfect competition!

• Realistic? Price wars between PC makers
3 Second-price Auction

- Nicholson, Ch. 18, pp. 669–676

- Sealed-bid auction

- Highest bidder wins object

- Price paid is second highest price

- Two individuals: $I = 2$

- Strategy $s_i$ is bid $b_i$

- Each individual knows value $v_i$
Payoff for individual $i$ is

$$u_i(b_i, b_{-i}) = \begin{cases} 
  v_i - b_i & \text{if } b_i > b_{-i} \\
  \frac{(v_i - b_{-i})}{2} & \text{if } b_i = b_{-i} \\
  0 & \text{if } b_i < b_{-i}
\end{cases}$$

Show: weakly dominant to set $b_i^* = v_i$

To show:

$$u_i(v_i, b_{-i}) \geq u_i(b_i, b_{-i})$$

for all $b_i$, for all $b_{-i}$, and for $i = 1, 2$. 
1. Assume $b_{-i} > v_i$
   
   - $u_i(v_i, b_{-i}) = 0 = u_i(b_i, b_{-i})$ for any $b_i < b_{-i}$
   - $u_i(b_{-i}, b_{-i}) = (v_i - b_{-i}) / 2 < 0$
   - $u_i(b_i, b_{-i}) = (v_i - b_{-i}) < 0$ for any $b_i > b_{-i}$

2. Assume now $b_{-i} = v_i$
3. Assume now \( b_{-i} < v_i \)
4 Auctions: Evidence from eBay

• In second-price auction, optimal strategy is to bid one’s own value

• Is this true?

• eBay has proxy system: If you have highest bid, you pay bid of second-highest bidder

• eBay is essentially a second-price auction

• Two deviations:
  1. People bid multiple times – they should not in this theory
  2. People may overbid
An example: eBay Bidding for a Board Game

- Bidding environment with clear boundary for rational willingness to pay ("buy-it-now price").
- Empirical environment unaffected by common-value arguments (presumably bidding for private use; in addition "buy-it-now" price).
- Still non-negligible amount ($100-$200).

→ Is there evidence of overbidding?
→ If so, can we detect determinants of overbidding?
The Object
The Data

- Cashflow 101: board game with the purpose of finance/accounting education.
- Retail price: $195 plus shipping cost ($10.75) from manufacturer (www.richdad.com).
- Two ways to purchase Cashflow 101 on eBay
  - Auction (quasi-second price proxy bidding)
  - Buy-it-now
Sample

- **Listings**
  - 206 by individuals (187 auctions only, 19 auctions with buy-it-now option)
  - 493 by two retailers (only buy-it-now)

- **Remove non-US$, terminated, unsold items and items without simultaneous professional buy-it-now listing.** → 169 auctions

- **Buy-it-now offers of the two retailers**
  - Continuously present for all but six days. (Often individual buy-it-now offers present as well; they are often lower.)
  - 100% and 99.9% positive feedback scores.
  - Same prices $129.95 until 07/31/2004; $139.95 since 08/01/2004.
  - Shipping cost $9.95; other retailer $10.95.
  - New items (with bonus tapes/video).
<table>
<thead>
<tr>
<th>Item</th>
<th>Quantity</th>
<th>Price</th>
<th>Delivery Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich Dad's Cashflow Quadrant, Rich dad ...</td>
<td></td>
<td>$12.50</td>
<td>1d 00h 14m</td>
</tr>
<tr>
<td>Rich Dad's Cashflow Quadrant by Robert T. ...</td>
<td></td>
<td>$9.00</td>
<td>1d 00h 43m</td>
</tr>
<tr>
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<td>1d 04h 36m</td>
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<tr>
<td>CASHFLOW® 101 202 Robert Kiyosaki Best Pak $</td>
<td></td>
<td>$207.96</td>
<td>1d 06h 47m</td>
</tr>
<tr>
<td>TRY IT TODAY, WITH ABSOLUTELY NO RISK,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CASHFLOW® 101 Robert Kiyosaki Plus Bonuses!</td>
<td></td>
<td>$129.95</td>
<td>1d 08h 02m</td>
</tr>
<tr>
<td>Your satisfaction is GUARANTEED, 100% $ back</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MINT Cashflow 101 *Robert Kiyosaki Game NR!</td>
<td></td>
<td>$140.00</td>
<td>1d 08h 04m</td>
</tr>
<tr>
<td>It's easy to be rich. Brand New. Still sealed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cashflow Hard Money Funding 101 real estate</td>
<td></td>
<td>$14.99</td>
<td>1d 09h 28m</td>
</tr>
<tr>
<td>BRANDNEW RICHDAD CASHFLOW FOR KIDS E-GAME</td>
<td></td>
<td>$20.00</td>
<td>1d 13h 54m</td>
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<tr>
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<td>$129.95</td>
<td>1d 14h 17m</td>
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<td>Your satisfaction is GUARANTEED, 100% $ back</td>
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</tr>
<tr>
<td>CASHFLOW® 101 202 Robert Kiyosaki Best Pak $</td>
<td></td>
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<td>1d 15h 47m</td>
</tr>
<tr>
<td>TRY IT TODAY, WITH ABSOLUTELY NO RISK,</td>
<td></td>
<td></td>
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</table>
Listing Example – Magnified

CASHFLOW® 101 202 Robert Kiyosaki Best Pak $207.96
TRY IT TODAY, WITH ABSOLUTELY NO RISK,

CASHFLOW® 101 Robert Kiyosaki Plus Bonuses! $129.95
Your satisfaction is GUARANTEED, 100% $ back

MINT Cashflow 101 *Robert Kiyosaki Game NR! $140.00
It's easy to be rich. Brand New. Still sealed

Pricing:
[Buy Now]
$129.95

Pricing:
$140.00
Bidding history of an item

<table>
<thead>
<tr>
<th>User ID</th>
<th>Bid Amount</th>
<th>Date of bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>breezebugs (21)</td>
<td>US $152.50</td>
<td>Aug-11-04 09:51:21 PDT</td>
</tr>
<tr>
<td>midipax (21)</td>
<td>US $150.00</td>
<td>Aug-11-04 09:39:53 PDT</td>
</tr>
<tr>
<td>breezebugs (21)</td>
<td>US $140.00</td>
<td>Aug-08-04 12:00:05 PDT</td>
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<tr>
<td>sj_orbit (86)</td>
<td>US $130.01</td>
<td>Aug-08-04 23:49:02 PDT</td>
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<tr>
<td>successbroker (931)</td>
<td>US $110.00</td>
<td>Aug-08-04 19:56:26 PDT</td>
</tr>
<tr>
<td>successbroker (931)</td>
<td>US $105.00</td>
<td>Aug-06-04 17:19:21 PDT</td>
</tr>
<tr>
<td>002la (1)</td>
<td>US $102.50</td>
<td>Aug-06-04 17:11:31 PDT</td>
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<td>successbroker (931)</td>
<td>US $100.00</td>
<td>Aug-05-04 15:41:40 PDT</td>
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<td>002la (1)</td>
<td>US $96.00</td>
<td>Aug-06-04 17:10:21 PDT</td>
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<tr>
<td>12-gauge (23)</td>
<td>US $88.00</td>
<td>Aug-05-04 09:13:30 PDT</td>
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<td>limyque (110)</td>
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<tr>
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<td>75con (1)</td>
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<td>Aug-05-04 06:43:26 PDT</td>
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<tr>
<td>bearnibulls22 (3)</td>
<td>US $25.00</td>
<td>Aug-05-04 06:48:01 PDT</td>
</tr>
</tbody>
</table>

If you and another bidder placed the same bid amount, the earlier bid takes priority.
Hypotheses

Given the information on the listing website:
• (H1) An auction should never end at a price above the concurrently available purchase price.
• (H2) Mentioning of higher outside prices should not affect bidding behavior.
Figure 1. Starting Price *(startprice)*

- 45% below $20; mean=$46; SD=43.88
- only 6 auctions with first bid (not price) above buy-it-now
Figure 2. Final Price (finalprice)
⇒ 41% are above “buy-it-now” (mean $132; SD 16.83)
Figure 4. Total Price (incl. shipping cost)

→ 51% are above “buy-it-now” plus its shipping cost (mean=$144.20; SD=15.00)
Dynamic Games

- Nicholson, Ch. 8, pp. 268-277

- Dynamic games: one player plays after the other

- Decision trees
  - Decision nodes
  - Strategy is a plan of action at each decision node
• Example: battle of the sexes game

<table>
<thead>
<tr>
<th></th>
<th>She \ He</th>
<th>Ballet</th>
<th>Football</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballet</td>
<td>2, 1</td>
<td>0, 0</td>
<td></td>
</tr>
<tr>
<td>Football</td>
<td>0, 0</td>
<td>1, 2</td>
<td></td>
</tr>
</tbody>
</table>

• Dynamic version: she plays first
• **Subgame-perfect equilibrium.** At each node of the tree, the player chooses the strategy with the highest payoff, given the other players’ strategy.

• Backward induction. Find optimal action in last period and then work backward.

• Solution
• Example 2: Entry Game

<table>
<thead>
<tr>
<th></th>
<th>Enter</th>
<th>Do not Enter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enter</td>
<td>$-1, -1$</td>
<td>$10, 0$</td>
</tr>
<tr>
<td>Do not Enter</td>
<td>$0, 5$</td>
<td>$0, 0$</td>
</tr>
</tbody>
</table>

• Exercise. Dynamic version.

• Coordination games solved if one player plays first
• Can use this to study finitely repeated games

• Suppose we play the prisoner’s dilemma game ten times.

\[
\begin{array}{c|ccc}
1 & 2 & D & ND \\
\hline
D & -4, -4 & -1, -5 \\
ND & -5, -1 & -2, -2 \\
\end{array}
\]

• What is the subgame perfect equilibrium?
• The result differs if infinite repetition with a probability of terminating

• Can have cooperation

• Strategy of repeated game:
  – Cooperate (ND) as long as opponent always cooperate
  – Defect (D) forever after first defection

• Theory of repeated games: Econ. 104
6 Next lecture

- General Equilibrium
- Barter