Economics 101A
(Lecture 21)

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Outline

1. Game Theory

2. Oligopoly: Cournot

3. Oligopoly: Bertrand
1 Game Theory

• Nicholson, Ch. 8, pp. 251-268

• Unfortunate name

• Game theory: study of decisions when payoff of player $i$ depends on actions of player $j$.

• Brief history:
  
  – von Neuman and Morgenstern, Theory of Games and Economic Behavior (1944)

  – Nash, Non-cooperative Games (1951)

  – ...

  – Nobel Prize to Nash, Harsanyi (Berkeley), Selten (1994)
• Definitions:

  – Players: $1, \ldots, I$

  – Strategy $s_i \in S_i$

  – Payoffs: $U_i(s_i, s_{-i})$
• Example: Prisoner’s Dilemma

- $I = 2$

- $s_i = \{D, ND\}$

- Payoffs matrix:

\[
\begin{array}{ccc}
1 & 2 & D & ND \\
D & -4, -4 & -1, -5 \\
ND & -5, -1 & -2, -2 \\
\end{array}
\]
• What prediction?

• Maximize sum of payoffs?

• Choose dominant strategies

• **Equilibrium in dominant strategies**

  • Strategies $s^* = (s_i^*, s_{-i}^*)$ are an Equilibrium in dominant strategies if

    $$U_i(s_i^*, s_{-i}) \geq U_i(s_i, s_{-i})$$

    for all $s_i \in S_i$, for all $s_{-i} \in S_{-i}$ and all $i = 1, \ldots, I$
• Battle of the Sexes game:

<table>
<thead>
<tr>
<th></th>
<th>Ballet</th>
<th>Football</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballet</td>
<td>2, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>Football</td>
<td>0, 0</td>
<td>1, 2</td>
</tr>
</tbody>
</table>

• Choose dominant strategies? Do not exist

• Nash Equilibrium.

• Strategies \( s^* = (s_i^*, s_{-i}^*) \) are a Nash Equilibrium if

\[
U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*)
\]

for all \( s_i \in S_i \) and \( i = 1, \ldots, I \)
• Is Nash Equilibrium unique?

• Does it always exist?

• Penalty kick in soccer (matching pennies)

<table>
<thead>
<tr>
<th>Kicker \ Goalie</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0,1</td>
<td>1,0</td>
</tr>
<tr>
<td>R</td>
<td>1,0</td>
<td>0,1</td>
</tr>
</tbody>
</table>

• Equilibrium always exists in mixed strategies $\sigma$
• Mixed strategy: allow for probability distribution.

• Back to penalty kick:

  – Kicker kicks left with probability $k$
  
  – Goalie kicks left with probability $g$

  – utility for kicker of playing $L$:

    $$U_K(L, \sigma) = gU_K(L, L) + (1 - g)U_K(L, R)$$
    $$= (1 - g)$$

  – utility for kicker of playing $R$:

    $$U_K(R, \sigma) = gU_K(R, L) + (1 - g)U_K(R, R)$$
    $$= g$$
Optimum?

- $L \succ R$ if $1 - g > g$ or $g < 1/2$
- $R \succ L$ if $1 - g < g$ or $g > 1/2$
- $L \sim R$ if $1 - g = g$ or $g = 1/2$

- Plot best response for kicker

- Plot best response for goalie
Nash Equilibrium is:

- fixed point of best response correspondence

- crossing of best response correspondences
2 Oligopoly: Cournot

- Nicholson, Ch. 15, pp. 534-540

- Back to oligopoly maximization problem

- Assume 2 firms, cost $c_i(y_i) = cy_i$, $i = 1, 2$

- Firms choose simultaneously quantity $y_i$

- Firm $i$ maximizes:

\[
\max_{y_i} p (y_i + y_{-i}) y_i - cy_i.
\]

- First order condition with respect to $y_i$:

\[
p'_{Y} (y_i^* + y_{-i}^*) y_i^* + p - c = 0, \ i = 1, 2.
\]
- Nash equilibrium:
  - \( y_1 \) optimal given \( y_2 \);
  - \( y_2 \) optimal given \( y_1 \).

- Solve equations:

\[
p_Y \left( y_1^* + y_2^* \right) y_1^* + p - c = 0 \quad \text{and} \quad p_Y \left( y_2^* + y_1^* \right) y_2^* + p - c = 0.
\]

- Cournot \( \rightarrow \) Pricing above marginal cost

- Numerical example \( \rightarrow \) Problem set 5
3 Oligopoly: Bertrand

- Nicholson, Ch. 15, pp. 533-534

- Cournot oligopoly: firms choose quantities

- Bertrand oligopoly: firms first choose prices, and then produce quantity demanded by market

- Market demand function $Y(p)$

- 2 firms

- Profits:

$$\pi_i(p_i, p_{-i}) = \begin{cases} 
(p_i - c)Y(p_i) & \text{if } p_i < p_{-i} \\
(p_i - c)Y(p_i)/2 & \text{if } p_i = p_{-i} \\
0 & \text{if } p_i > p_{-i} 
\end{cases}$$
• First show that $p_1 = c = p_2$ is Nash Equilibrium

• Does any firm have a (strict) incentive to deviate?

• Check profits for Firm 1

• Symmetric argument for Firm 2
• Second, show that this equilibrium is unique.

• For each of the next 5 cases at least on firm has a profitable deviation

• Case 1. $p_1 > p_2 > c$

• Case 2. $p_1 = p_2 > c$

• Case 3. $p_1 > c \geq p_2$
• Case 4. \( c > p_1 \geq p_2 \)

• Case 5. \( p_1 = c > p_2 \)

• Only Case 6 remains: \( p_1 = c = p_2 \), which is Nash Equilibrium

• It is unique!
• Notice:

• To show that something is an equilibrium $\rightarrow$ Show that there is *no* profitable deviation

• To show that something is *not* an equilibrium $\rightarrow$ Show that there is *one* profitable deviation
• Surprising result of Bertrand Competition

• Marginal cost pricing

• Two firms are enough to guarantee perfect competition!

• Realistic? Price wars between PC makers
4 Next lecture

- Dynamic Games

- Stackelberg duopoly