Outline

1. Cost Curves II

2. One-step Profit Maximization

3. Second-Order Conditions

4. Introduction to Market Equilibrium

5. Aggregation

6. Market Equilibrium in the Short-Run
1 Cost Curves II

- **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

- **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?
2 One-step Profit Maximization

- Nicholson, Ch. 11, pp. 383-393

- One-step procedure: maximize profits

- Perfect competition. Price $p$ is given
  - Firms are small relative to market
  - Firms do not affect market price $p_M$

  - Will firm produce at $p > p_M$?
  - Will firm produce at $p < p_M$?
  - $\implies p = p_M$
- Revenue: $py = pf(L, K)$

- Cost: $wL + rK$

- Profit $pf(L, K) - wL - rK$
• Agent optimization:

$$\max_{L,K} pf(L, K) - wL - rK$$

• First order conditions:

$$pf'_L(L, K) - w = 0$$

and

$$pf'_K(L, K) - r = 0$$

• Second order conditions? $pf''_{L,L}(L, K) < 0$ and

$$|H| = \begin{vmatrix} pf''_{L,L}(L, K) & pf''_{L,K}(L, K) \\ pf''_{L,K}(L, K) & pf''_{K,K}(L, K) \end{vmatrix} =$$

$$= p^2 \left[ f''_{L,L}f''_{K,K} - (f''_{L,K})^2 \right] > 0$$

• Need $f''_{L,K}$ not too large for maximum
• Comparative statics with respect to to $p$, $w$, and $r$.

• What happens if $w$ increases?

\[
\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & p_{f_{L,K}}''(L,K) \\ 0 & p_{f_{K,K}}''(L,K) \end{vmatrix}}{\begin{vmatrix} p_{f_{L,L}}''(L,K) & p_{f_{L,K}}''(L,K) \\ p_{f_{L,K}}''(L,K) & p_{f_{K,K}}''(L,K) \end{vmatrix}} < 0
\]

and

\[
\frac{\partial L^*}{\partial r} =
\]

• Sign of $\partial L^*/\partial r$ depends on $f_{L,K}''$. 

3 Second Order Conditions in P-Max: Cobb-Douglas

• How do the second order conditions relate for:
  – Cost Minimization
  – Profit Maximization?

• Check for Cobb-Douglas production function
  \[ y = AK^\alpha L^\beta \]

• Cost Minimization. S.o.c.:
  \[ c''_y (y^*, w, r) > 0 \]

• As we showed, for CD prod. ftn.,
  \[ c''_y (y^*, w, r) = -\frac{1}{\alpha + \beta} \frac{1 - (\alpha + \beta)}{\alpha + \beta} \frac{B}{A^2} \left( \frac{y}{A} \right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}} \]
  which is $> 0$ as long as $\alpha + \beta < 1$ (DRS)
• **Profit Maximization.** S.o.c.:

\[ p f''_{L,L}(L, K) < 0 \]

and

\[
|H| = p^2 \left[ f''_{L,L} f''_{K,K} - (f''_{L,K})^2 \right] > 0
\]

• As long as \( \beta < 1 \),

\[
p f''_{L,L} = p \beta (\beta - 1) AK^\alpha L^\beta - 2 < 0
\]

• Then,

\[
|H| = p^2 \left[ f''_{L,L} f''_{K,K} - (f''_{L,K})^2 \right] =
\]

\[
= p^2 \left[ \beta (\beta - 1) AK^\alpha L^{\beta - 2} \right] =
\]

\[
= \alpha (\alpha - 1) AK^{\alpha - 2} L^{\beta - 2} \]

\[
= p^2 A^2 K^{2\alpha - 2} L^{2\beta - 2} \alpha \beta \left[ 1 - \alpha - \beta \right]
\]

• Therefore, \( |H| > 0 \) iff \( \alpha + \beta < 1 \) (DRS)

• The two conditions coincide
4 Introduction to Market Equilibrium

- Nicholson, Ch. 12, pp. 409–419

- Two ways to analyze firm behavior:
  - Two-Step Cost Minimization
  - One-Step Profit Maximization

- What did we learn?
  - Optimal demand for inputs $L^*$, $K^*$ (see above)
  - Optimal quantity produced $y^*$
• Supply function. $y = y^* (p, w, r)$

  – From profit maximization:
    $$y = f (L^* (p, w, r), K^* (p, w, r))$$

  – From cost minimization:
    $$MC \text{ curve above } AC$$

  – Supply function is increasing in $p$

• Market Equilibrium. Equate demand and supply.

• Aggregation?

• Industry supply function!
5 Aggregation

5.1 Producers aggregation

- $J$ companies, $j = 1, \ldots, J$, producing good $i$

- Company $j$ has supply function

\[ y_i^j = y_i^{j*}(p_i, w, r) \]

- Industry supply function:

\[ Y_i(p_i, w, r) = \sum_{j=1}^{J} y_i^{j*}(p_i, w, r) \]

- Graphically,
5.2 Consumer aggregation

- *One-consumer economy*

- Utility function $u(x_1, \ldots, x_n)$

- Prices $p_1, \ldots, p_n$

- Maximization $\implies$

\[
x_1^* = x_1^*(p_1, \ldots, p_n, M),
\]

\[
\vdots
\]

\[
x_n^* = x_n^*(p_1, \ldots, p_n, M).
\]
• Focus on good $i$. Fix prices $p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$ and $M$

• **Single-consumer demand function:**

  $x^*_i = x^*_i (p_i | p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n, M)$

• What is sign of $\partial x^*_i / \partial p_i$?

  • Negative if good $i$ is normal

  • Negative or positive if good $i$ is inferior
• **Aggregation:** \( J \) consumers, \( j = 1, \ldots, J \)

• Demand for good \( i \) by consumer \( j \):

\[
x^{j*}_i = x^{j*}_i (p_1, \ldots, p_n, M^j)
\]

• Market demand \( X_i \):

\[
X_i (p_1, \ldots, p_n, M^1, \ldots, M^J) = \sum_{j=1}^{J} x^{j*}_i (p_1, \ldots, p_n, M^j)
\]

• Graphically,
• Notice: market demand function depends on distribution of income $M^J$

• Market demand function $X_i$:
  - Consumption of good $i$ as function of prices $p$
  - Consumption of good $i$ as function of income distribution $M^J$
6 Market Equilibrium in the Short-Run

- What is equilibrium price $p_i$?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $p^*$ equates demand and supply of good $i$:

$$Y^* = Y^S_i (p^*_i, w, r) = X^D_i (p^*_1, ..., p^*_n, M^1, ..., M^J)$$
• Graphically,

• Notice: in short-run firms can make positive profits
Comparative statics exercises with endogenous price $p_i$:

- increase in wage $w$ or interest rate $r$:

- change in income distribution
7 Next Lecture

- Market Equilibrium

- Comparative Statics of Equilibrium

- Elasticities

- Taxes and Subsidies