

# Economics 101A

## (Lecture 16)

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March 14, 2017

## Outline

1. Cost Curves II
2. One-step Profit Maximization
3. Second-Order Conditions
4. Introduction to Market Equilibrium
5. Aggregation
6. Market Equilibrium in the Short-Run



## 2 One-step Profit Maximization

- Nicholson, Ch. 11, pp. 383-393
- One-step procedure: maximize profits
- Perfect competition. Price  $p$  is given
  - Firms are small relative to market
  - Firms do not affect market price  $p_M$
  - Will firm produce at  $p > p_M$ ?
  - Will firm produce at  $p < p_M$ ?
  - $\implies p = p_M$

- Revenue:  $py = pf(L, K)$
- Cost:  $wL + rK$
- Profit  $pf(L, K) - wL - rK$

- Agent optimization:

$$\max_{L,K} pf(L, K) - wL - rK$$

- First order conditions:

$$pf'_L(L, K) - w = 0$$

and

$$pf'_K(L, K) - r = 0$$

- Second order conditions?  $pf''_{L,L}(L, K) < 0$  and

$$\begin{aligned} |H| &= \begin{vmatrix} pf''_{L,L}(L, K) & pf''_{L,K}(L, K) \\ pf''_{L,K}(L, K) & pf''_{K,K}(L, K) \end{vmatrix} = \\ &= p^2 \left[ f''_{L,L} f''_{K,K} - (f''_{L,K})^2 \right] > 0 \end{aligned}$$

- Need  $f''_{L,K}$  not too large for maximum

- Comparative statics with respect to  $p$ ,  $w$ , and  $r$ .
- What happens if  $w$  increases?

$$\frac{\partial L^*}{\partial w} = - \frac{\begin{vmatrix} -1 & pf''_{L,K}(L, K) \\ 0 & pf''_{K,K}(L, K) \end{vmatrix}}{\begin{vmatrix} pf''_{L,L}(L, K) & pf''_{L,K}(L, K) \\ pf''_{L,K}(L, K) & pf''_{K,K}(L, K) \end{vmatrix}} < 0$$

and

$$\frac{\partial L^*}{\partial r} =$$

- Sign of  $\partial L^* / \partial r$  depends on  $f''_{L,K}$ .

### 3 Second Order Conditions in P-Max: Cobb-Douglas

- How do the second order conditions relate for:
  - Cost Minimization
  - Profit Maximization?

- Check for Cobb-Douglas production function

$$y = AK^\alpha L^\beta$$

- **Cost Minimization.** S.o.c.:

$$c''_y(y^*, w, r) > 0$$

- As we showed, for CD prod. ftn.,

$$c''_y(y^*, w, r) = -\frac{1}{\alpha + \beta} \frac{1 - (\alpha + \beta)}{\alpha + \beta} \frac{B}{A^2} \left(\frac{y}{A}\right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}}$$

which is  $> 0$  as long as  $\alpha + \beta < 1$  (DRS)



- **Profit Maximization.** S.o.c.:  $pf''_{L,L}(L, K) < 0$   
and

$$|H| = p^2 \left[ f''_{L,L} f''_{K,K} - (f''_{L,K})^2 \right] > 0$$

- As long as  $\beta < 1$ ,

$$pf''_{L,L} = p\beta(\beta - 1)AK^\alpha L^{\beta-2} < 0$$

- Then,

$$\begin{aligned} |H| &= p^2 \left[ f''_{L,L} f''_{K,K} - (f''_{L,K})^2 \right] = \\ &= p^2 \left[ \begin{array}{c} \beta(\beta - 1)AK^\alpha L^{\beta-2} \\ \alpha(\alpha - 1)AK^{\alpha-2}L^\beta \\ (\alpha\beta AK^{\alpha-1}L^{\beta-1})^2 \end{array} \right] = \\ &= p^2 A^2 K^{2\alpha-2} L^{2\beta-2} \alpha\beta [1 - \alpha - \beta] \end{aligned}$$

- Therefore,  $|H| > 0$  iff  $\alpha + \beta < 1$  (DRS)
- The two conditions coincide

# 4 Introduction to Market Equilibrium

- Nicholson, Ch. 12, pp. 409–419
- Two ways to analyze firm behavior:
  - Two-Step Cost Minimization
  - One-Step Profit Maximization
- What did we learn?
  - Optimal demand for inputs  $L^*$ ,  $K^*$  (see above)
  - Optimal quantity produced  $y^*$

- **Supply function.**  $y = y^*(p, w, r)$

- From profit maximization:

$$y = f(L^*(p, w, r), K^*(p, w, r))$$

- From cost minimization:

*MC* curve above *AC*

- Supply function is increasing in  $p$

- Market Equilibrium. Equate demand and supply.

- Aggregation?

- Industry supply function!

# 5 Aggregation

## 5.1 Producers aggregation

- $J$  companies,  $j = 1, \dots, J$ , producing good  $i$
- Company  $j$  has supply function

$$y_i^j = y_i^{j*}(p_i, w, r)$$

- Industry supply function:

$$Y_i(p_i, w, r) = \sum_{j=1}^J y_i^{j*}(p_i, w, r)$$

- Graphically,

## 5.2 Consumer aggregation

- *One-consumer economy*
- Utility function  $u(x_1, \dots, x_n)$
- prices  $p_1, \dots, p_n$
- Maximization  $\implies$

$$\begin{aligned}x_1^* &= x_1^*(p_1, \dots, p_n, M), \\ &\vdots \\ x_n^* &= x_n^*(p_1, \dots, p_n, M).\end{aligned}$$

- Focus on good  $i$ . Fix prices  $p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_n$  and  $M$

- **Single-consumer demand function:**

$$x_i^* = x_i^*(p_i | p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_n, M)$$

- What is sign of  $\partial x_i^* / \partial p_i$ ?
- Negative if good  $i$  is normal
- Negative or positive if good  $i$  is inferior

- *Aggregation*:  $J$  consumers,  $j = 1, \dots, J$

- Demand for good  $i$  by consumer  $j$  :

$$x_i^{j*} = x_i^{j*} (p_1, \dots, p_n, M^j)$$

- Market demand  $X_i$ :

$$\begin{aligned} X_i & (p_1, \dots, p_n, M^1, \dots, M^J) \\ &= \sum_{j=1}^J x_i^{j*} (p_1, \dots, p_n, M^j) \end{aligned}$$

- Graphically,

- Notice: market demand function depends on distribution of income  $M^J$
  
- Market demand function  $X_i$ :
  - Consumption of good  $i$  as function of prices  $\mathbf{p}$
  - Consumption of good  $i$  as function of income distribution  $M^j$



## 6 Market Equilibrium in the Short-Run

- What is equilibrium price  $p_i$ ?
- Magic of the Market...
- Equilibrium: No excess supply, No excess demand
- Prices  $\mathbf{p}^*$  equates demand and supply of good  $i$ :

$$Y^* = Y_i^S(p_i^*, w, r) = X_i^D(p_1^*, \dots, p_n^*, M^1, \dots, M^J)$$

- Graphically,

- Notice: in short-run firms can make positive profits

- Comparative statics exercises with endogenous price

$p_i$  :

- increase in wage  $w$  or interest rate  $r$ :

- change in income distribution

# 7 Next Lecture

- Market Equilibrium
- Comparative Statics of Equilibrium
- Elasticities
- Taxes and Subsidies