Outline

1. Time Consistency
2. Time Inconsistency
3. Health Club Attendance
4. Production: Introduction
5. Production Function
1 Time consistency

• Intertemporal choice

• Three periods, $t = 0$, $t = 1$, and $t = 2$

• At each period $i$, agents:

  – have income $M_i' = M_i + \text{savings/debts from previous period}$

  – choose consumption $c_i$;

  – can save/borrow $M_i' - c_i$

  – no borrowing in last period: at $t = 2$ $M_2' = c_2$
• Utility function at $t = 0$

$$u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1 + \delta} U(c_1) + \frac{1}{(1 + \delta)^2} U(c_2)$$

• Utility function at $t = 1$

$$u(c_1, c_2) = U(c_1) + \frac{1}{1 + \delta} U(c_2)$$

• Utility function at $t = 2$

$$u(c_2) = U(c_2)$$

• $U' > 0$, $U'' < 0$
• Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

• Period 1.

• Budget constraint at $t = 1$:

$$c_1 + \frac{1}{1 + r}c_2 \leq M'_1 + \frac{1}{1 + r}M_2$$

• Maximization problem:

$$\max U(c_1) + \frac{1}{1 + \delta}U(c_2)$$

$$s.t. \ c_1 + \frac{1}{1 + r}c_2 \leq M'_1 + \frac{1}{1 + r}M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U'(c_1)}{U'(c_2)} = \frac{1 + r}{1 + \delta}$$
• Back to period 0.

• Agent at time 0 can commit to consumption at time 1 as function of uncertain income $M_1$.

• Anticipated budget constraint at $t = 1$:

$$c_1 + \frac{1}{1 + r}c_2 \leq M'_1 + \frac{1}{1 + r}M_2$$

• Maximization problem:

$$\max U(c_0) + \frac{1}{1 + \delta}U(c_1) + \frac{1}{(1 + \delta)^2}U(c_2)$$

$$s.t. \ c_1 + \frac{1}{1 + r}c_2 \leq M'_1 + \frac{1}{1 + r}M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U''(c_1)}{U''(c_2)} = \frac{1 + r}{1 + \delta}$$
• The two conditions coincide!

• **Time consistency.** Plans for future coincide with future actions.

• To see why, rewrite utility function $u(c_0, c_1, c_2)$:

$$U(c_0) + \frac{1}{1 + \delta} U(c_1) + \frac{1}{(1 + \delta)^2} U(c_2)$$

$$= U(c_0) + \frac{1}{1 + \delta} \left[ U(c_1) + \frac{1}{1 + \delta} U(c_2) \right]$$

• Expression in brackets coincides with utility at $t = 1$

• Is time consistency right?

  – addictive products (alcohol, drugs);
  
  – good actions (exercising, helping friends);
  
  – immediate gratification (shopping, credit card borrowing)
2 Time Inconsistency

• Alternative specification (Akerlof, 1991; Laibson, 1997; O’Donoghue and Rabin, 1999)

• Utility at time \( t \) is \( u(c_t, c_{t+1}, c_{t+2}) : \)

\[
  u(c_t) + \frac{\beta}{1 + \delta} u(c_{t+1}) + \frac{\beta}{(1 + \delta)^2} u(c_{t+2}) + \ldots
\]

• Discount factor is

\[
  1, \frac{\beta}{1 + \delta'}, \frac{\beta}{(1 + \delta)^2}, \frac{\beta}{(1 + \delta)^3}, \ldots
\]

instead of

\[
  1, \frac{1}{1 + \delta'}, \frac{1}{(1 + \delta)^2}, \frac{1}{(1 + \delta)^3}, \ldots
\]

• What is the difference?

• Immediate gratification: \( \beta < 1 \)
Back to our problem: **Period 1.**

Maximization problem:

$$\max U(c_1) + \frac{\beta}{1 + \delta} U(c_2)$$

subject to:

$$c_1 + \frac{1}{1 + r} c_2 \leq M_1' + \frac{1}{1 + r} M_2$$

First order conditions:

Ratio of f.o.c.s:

$$\frac{U'(c_1^*)}{U'(c_2^*)} = \beta \frac{1 + r}{1 + \delta}$$
• Now, period 0 with commitment.

• Maximization problem:

$$\max U(c_0) + \frac{\beta}{1 + \delta} U(c_1) + \frac{\beta}{(1 + \delta)^2} U(c_2)$$

s.t. $c_1 + \frac{1}{1 + r} c_2 \leq M'_1 + \frac{1}{1 + r} M_2$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U'(c^*_1, c)}{U'(c^*_2, c)} = \frac{1 + r}{1 + \delta}$$

• The two conditions differ!

• Time inconsistency: $c^*_{1,c} < c^*_1$ and $c^*_{2,c} > c^*_2$

• The agent allows him/herself too much immediate consumption and saves too little
• Ok, we agree. but should we study this as economists?

• YES!

  – One trillion dollars in credit card debt;

  – Most debt is in teaser rates;

  – Two thirds of Americans are overweight or obese;

  – $10bn health-club industry

• Is this testable?

  – In the laboratory?

  – In the field?
3 Health Club Attendance


- 3 health clubs

- Data on attendance from swiping cards

- Choice of contracts:
  - Monthly contract with average price of $75
  - 10-visit pass for $100

- Consider users that choose monthly contract. Attendance?
<table>
<thead>
<tr>
<th>Sample: No subsidy, all clubs</th>
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<tr>
<td>Average price per month</td>
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<td>Users initially enrolled with a monthly contract</td>
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<td>Month 1</td>
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<td>Months 1 to 6</td>
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<td>Users initially enrolled with an annual contract, who joined at least 14 months before the end of sample period</td>
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<td>Year 1</td>
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- Attend on average 4.8 times per month
- Pay on average over $17
- Average delay of 2.2 months ($185) between last attendance and contract termination

- Over membership, user could have saved $700 by paying per visit
• Health club attendance:
  – immediate cost $c$
  – delayed benefit $b$

• At sign-up (attend tomorrow):

\[ NB^t = -\frac{\beta}{1 + \delta}c + \frac{\beta}{(1 + \delta)^2}b \]

• Plan to attend if $NB^t > 0$

\[ c < \frac{1}{(1 + \delta)}b \]
\begin{itemize}
  \item Once moment to attend comes:
    \[ NB = -c + \frac{\beta}{(1 + \delta)}b \]
  \item Attend if \( NB > 0 \)
    \[ c < \frac{\beta}{(1 + \delta)}b \]
\end{itemize}
• Interpretations?

• Users are buying a commitment device

• User underestimate their future self-control problems:
  – They overestimate future attendance
  – They delay cancellation
4 Production: Introduction

• Second half of the economy. Production

• Example. Ford and the Minivan (Petrin, 2002):
  – Ford had idea: "Mini/Max" (early '70s)
  – Did Ford produce it?
  – No!
  – Ford was worried of cannibalizing station wagon sector
  – Chrysler introduces Dodge Caravan (1984)
  – Chrysler: $1.5bn profits (by 1987)!
• Why need separate treatment?

• Perhaps firms maximize utility...

• ...we can be more precise:
  – Competition
  – Institutional structure
5 Production Function

- Nicholson, Ch. 9, pp. 303-310; 313-318

- Production function: \( y = f(z) \). Function \( f : R_+^n \to R_+ \)

- Inputs \( z = (z_1, z_2, ..., z_n) \): labor, capital, land, human capital

- Output \( y \): Minivan, Intel CPU, mangoes (Philippines)

- Properties of \( f \):
  - no free lunches: \( f(0) = 0 \)
  - positive marginal productivity: \( f'_i(z) > 0 \)
  - decreasing marginal productivity: \( f''_{i,i}(z) < 0 \)
• Isoquants $Q(y) = \{x | f(x) = y\}$

• Set of inputs $z$ required to produce quantity $y$

• Special case. Two inputs:
  
  $- z_1 = L$ (labor)

  $- z_2 = K$ (capital)

• Isoquant: $f(L, K) - y = 0$

• Slope of isoquant $dK/dL = MRTS$
• Convex production function if convex isoquants

• Reasonable: combine two technologies and do better!

• Mathematically, convex isoquants if \( \frac{d^2K}{d^2L} > 0 \)

• Solution:

\[
\frac{d^2K}{d^2L} = -\frac{f''_{L,L}f'_K - 2f''_{L,K}f'_L + f''_{K,K} \left( f'_L \right)^2 / f'_K}{\left( f'_K \right)^2}
\]

• Hence, \( \frac{d^2K}{d^2L} > 0 \) if \( f''_{L,K} > 0 \) (inputs are complements in production)
6 Next Lecture

- Production

- Cost Minimization