Outline

1. Mid-Term Feedback

2. Risk Aversion and Lottery

3. Insurance

4. Investment in Risky Asset

5. Time Consistency
1 Mid-Term Feedback

- Thanks for the feedback!
2 Risk aversion and Lottery

• Risk aversion:
  – individuals dislike uncertainty
  – $u$ concave, $u'' < 0$

• Implications?
  – purchase of insurance (possible accident)
  – investment in risky asset (risky investment)
  – choice over time (future income uncertain)
• Experiment — Are you risk-averse?
3 Insurance

• Individual has:
  
  – wealth \( w \)
  
  – utility function \( u \), with \( u' > 0, u'' < 0 \)

• Probability \( p \) of accident with loss \( L \)

• Insurance offers coverage:
  
  – premium \( q \) for each \$1\ paid in case of accident
  
  – units of coverage purchased \( \alpha \)

• Individual maximization:

\[
\max_{\alpha} (1 - p) u (w - q\alpha) + pu (w - q\alpha - L + \alpha) \quad s.t. \alpha \geq 0
\]
• Assume $\alpha^* \geq 0$, check later

• First order conditions:

\[
0 = -q (1 - p) u' (w - q\alpha) + (1 - q) pu' (w - q\alpha - L + \alpha)
\]

or

\[
\frac{u' (w - q\alpha)}{u' (w - q\alpha - L + \alpha)} = \frac{1 - q}{q} \frac{p}{1 - p}.
\]

• Assume first $q = p$ (insurance is fair)

• Solution for $\alpha^* =$?
• $\alpha^* > 0$, so we are ok!

• What if $q > p$ (insurance needs to cover operating costs)?

• Insurance will be only partial (if at all): $\alpha^* < L$

• Exercise: Check second order conditions!
4 Investment in Risky Asset

• Individual has:
  – wealth \( w \)
  – utility function \( u \), with \( u' > 0 \)

• Two possible investments:
  – Asset B (bond) yields return 1 for each dollar
  – Asset S (stock) yields uncertain return \( (1 + r) \):
    * \( r = r_+ > 0 \) with probability \( p \)
    * \( r = r_- < 0 \) with probability \( 1 - p \)
    * \( E_r = pr_+ + (1 - p)r_- > 0 \)

• Share of wealth invested in stock \( S = \alpha \)
• Individual maximization:

\[
\max_{\alpha} (1 - p) u(w [(1 - \alpha) + \alpha (1 + r_-)]) +
+ pu(w [(1 - \alpha) + \alpha (1 + r_+)])
\]

\[s.t. 0 \leq \alpha \leq 1\]

• Case of risk neutrality: \(u(x) = a + bx, b > 0\)

• Assume \(a = 0\) (no loss of generality)

• Maximization becomes

\[
\max_{\alpha} b (1 - p) (w [1 + \alpha r_-]) + bp (w [1 + \alpha r_+])
\]

or

\[
\max_{\alpha} bw + \alpha bw [(1 - p) r_- + pr_+]
\]

• Sign of term in square brackets? Positive!

• Set \(\alpha^* = 1\)
• Case of risk aversion: \( u'' < 0 \)

• Assume \( 0 \leq \alpha^* \leq 1 \), check later

• First order conditions:

\[
0 = (1 - p) (wr_-) u' (w [1 + \alpha r_-]) + \\
+ p (wr_+) u' (w [1 + \alpha r_+])
\]

• Can \( \alpha^* = 0 \) be solution?

• Solution is \( \alpha^* > 0 \) (positive investment in stock)

• Exercise: Check s.o.c.
5 Time consistency

• Intertemporal choice

• Three periods, $t = 0$, $t = 1$, and $t = 2$

• At each period $i$, agents:
  
  – have income $M_i' = M_i + $savings/debts from previous period
  
  – choose consumption $c_i$;
  
  – can save/borrow $M_i' - c_i$
  
  – no borrowing in last period: at $t = 2$ $M_2' = c_2$
• Utility function at $t = 0$

$$u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1 + \delta} U(c_1) + \frac{1}{(1 + \delta)^2} U(c_2)$$

• Utility function at $t = 1$

$$u(c_1, c_2) = U(c_1) + \frac{1}{1 + \delta} U(c_2)$$

• Utility function at $t = 2$

$$u(c_2) = U(c_2)$$

• $U' > 0$, $U'' < 0$
• Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

• **Period 1.**

• Budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M'_1 + \frac{1}{1+r}M_2$$

• Maximization problem:

$$\max U(c_1) + \frac{1}{1+\delta}U(c_2)$$

$$s.t. \ c_1 + \frac{1}{1+r}c_2 \leq M'_1 + \frac{1}{1+r}M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U''(c_1)}{U''(c_2)} = \frac{1+r}{1+\delta}$$
• Back to **period 0**.

• Agent at time 0 can commit to consumption at time 1 as function of uncertain income $M_1$.

• Anticipated budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

• Maximization problem:

$$\max U(c_0) + \frac{1}{1+\delta}U(c_1) + \frac{1}{(1+\delta)^2}U(c_2)$$

$$s.t. c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

• First order conditions:

• Ratio of f.o.c.s:

$$\frac{U''(c_1)}{U''(c_2)} = \frac{1 + r}{1 + \delta}$$
• The two conditions coincide!

• **Time consistency.** Plans for future coincide with future actions.

• To see why, rewrite utility function $u(c_0, c_1, c_2)$:

  $$U(c_0) + \frac{1}{1 + \delta}U(c_1) + \frac{1}{(1 + \delta)^2}U(c_2)$$

  $$= U(c_0) + \frac{1}{1 + \delta} \left[ U(c_1) + \frac{1}{1 + \delta}U(c_2) \right]$$

• Expression in brackets coincides with utility at $t = 1$

• Is time consistency right?

  – addictive products (alcohol, drugs);

  – good actions (exercising, helping friends);

  – immediate gratification (shopping, credit card borrowing)
6 Next lecture and beyond

- Time Inconsistency

- Production Function