

Economics 101A

(Lecture 12)

Stefano DellaVigna

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Outline

1. Mid-Term Feedback
2. Risk Aversion and Lottery
3. Insurance
4. Investment in Risky Asset
5. Time Consistency

1 Mid-Term Feedback

- Thanks for the feedback!

2 Risk aversion and Lottery

- Risk aversion:
 - individuals dislike uncertainty
 - u concave, $u'' < 0$
- Implications?
 - purchase of insurance (possible accident)
 - investment in risky asset (risky investment)
 - choice over time (future income uncertain)

- Experiment — Are you risk-averse?

3 Insurance

- Individual has:
 - wealth w
 - utility function u , with $u' > 0$, $u'' < 0$
- Probability p of accident with loss L
- Insurance offers coverage:
 - premium $\$q$ for each $\$1$ paid in case of accident
 - units of coverage purchased α
- Individual maximization:

$$\begin{aligned} \max_{\alpha} & (1 - p) u(w - q\alpha) + pu(w - q\alpha - L + \alpha) \\ \text{s.t.} & \alpha \geq 0 \end{aligned}$$

- Assume $\alpha^* \geq 0$, check later

- First order conditions:

$$0 = -q(1-p)u'(w-q\alpha) + (1-q)pu'(w-q\alpha-L+\alpha)$$

or

$$\frac{u'(w-q\alpha)}{u'(w-q\alpha-L+\alpha)} = \frac{1-q}{q} \frac{p}{1-p}$$

- Assume first $q = p$ (insurance is fair)

- Solution for $\alpha^* = ?$

- $\alpha^* > 0$, so we are ok!
- What if $q > p$ (insurance needs to cover operating costs)?
- Insurance will be only partial (if at all): $\alpha^* < L$
- Exercise: Check second order conditions!

4 Investment in Risky Asset

- Individual has:
 - wealth w
 - utility function u , with $u' > 0$
- Two possible investments:
 - Asset B (bond) yields return 1 for each dollar
 - Asset S (stock) yields uncertain return $(1 + r)$:
 - * $r = r_+ > 0$ with probability p
 - * $r = r_- < 0$ with probability $1 - p$
 - * $Er = pr_+ + (1 - p)r_- > 0$
- Share of wealth invested in stock $S = \alpha$

- Individual maximization:

$$\begin{aligned} & \max_{\alpha} (1 - p) u(w [(1 - \alpha) + \alpha (1 + r_-)]) + \\ & + pu(w [(1 - \alpha) + \alpha (1 + r_+)]) \\ & s.t. 0 \leq \alpha \leq 1 \end{aligned}$$

- Case of risk neutrality: $u(x) = a + bx, b > 0$

- Assume $a = 0$ (no loss of generality)

- Maximization becomes

$$\max_{\alpha} b(1 - p)(w [1 + \alpha r_-]) + bp(w [1 + \alpha r_+])$$

or

$$\max_{\alpha} bw + \alpha bw [(1 - p) r_- + pr_+]$$

- Sign of term in square brackets? Positive!

- Set $\alpha^* = 1$

- Case of risk aversion: $u'' < 0$
- Assume $0 \leq \alpha^* \leq 1$, check later

- First order conditions:

$$0 = (1 - p)(wr_-)u'(w[1 + \alpha r_-]) + p(wr_+)u'(w[1 + \alpha r_+])$$

- Can $\alpha^* = 0$ be solution?

- Solution is $\alpha^* > 0$ (positive investment in stock)

- Exercise: Check s.o.c.

5 Time consistency

- Intertemporal choice
- Three periods, $t = 0$, $t = 1$, and $t = 2$
- At each period i , agents:
 - have income $M'_i = M_i + \text{savings/debts from previous period}$
 - choose consumption c_i ;
 - can save/borrow $M'_i - c_i$
 - no borrowing in last period: at $t = 2$ $M'_2 = c_2$

- Utility function at $t = 0$

$$u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1 + \delta} U(c_1) + \frac{1}{(1 + \delta)^2} U(c_2)$$

- Utility function at $t = 1$

$$u(c_1, c_2) = U(c_1) + \frac{1}{1 + \delta} U(c_2)$$

- Utility function at $t = 2$

$$u(c_2) = U(c_2)$$

- $U' > 0, U'' < 0$

- Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

- **Period 1.**

- Budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M'_1 + \frac{1}{1+r}M_2$$

- Maximization problem:

$$\begin{aligned} \max U(c_1) + \frac{1}{1+\delta}U(c_2) \\ s.t. c_1 + \frac{1}{1+r}c_2 \leq M'_1 + \frac{1}{1+r}M_2 \end{aligned}$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1)}{U'(c_2)} = \frac{1+r}{1+\delta}$$

- Back to **period 0**.
- Agent at time 0 can commit to consumption at time 1 as function of uncertain income M_1 .

- Anticipated budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M'_1 + \frac{1}{1+r}M_2$$

- Maximization problem:

$$\begin{aligned} \max & U(c_0) + \frac{1}{1+\delta}U(c_1) + \frac{1}{(1+\delta)^2}U(c_2) \\ \text{s.t.} & c_1 + \frac{1}{1+r}c_2 \leq M'_1 + \frac{1}{1+r}M_2 \end{aligned}$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1)}{U'(c_2)} = \frac{1+r}{1+\delta}$$

- The two conditions coincide!
- **Time consistency.** Plans for future coincide with future actions.

- To see why, rewrite utility function $u(c_0, c_1, c_2)$:

$$\begin{aligned}
 & U(c_0) + \frac{1}{1 + \delta}U(c_1) + \frac{1}{(1 + \delta)^2}U(c_2) \\
 = & U(c_0) + \frac{1}{1 + \delta} \left[U(c_1) + \frac{1}{1 + \delta}U(c_2) \right]
 \end{aligned}$$

- Expression in brackets coincides with utility at $t = 1$
- Is time consistency right?
 - addictive products (alcohol, drugs);
 - good actions (exercising, helping friends);
 - immediate gratification (shopping, credit card borrowing)

6 Next lecture and beyond

- Time Inconsistency
- Production Function