Outline

1. Slutsky Equation

2. Complements and substitutes

3. Application 1: Labor Supply
1 Slutsky Equation

• Nicholson, Ch. 5, pp. 160-163

• Now: go back to Utility Max. in case where $p_2$ increases to $p'_2 > p_2$

• What is $\partial x^*_2 / \partial p_2$? Decompose effect:

  1. Substitution effect of an increase in $p_i$
     - $\partial h^*_2 / \partial p_2$, that is change in EMIN point as $p_2$ decreases
     - Moving along an indifference curve
     - Certainly $\partial h^*_2 / \partial p_2 < 0$
2. Income effect of an increase in $p_i$

- $\partial x^*_2/\partial M$, increase in consumption of good 2 due to increased income

- Shift out a budget line

- $\partial x^*_2/\partial M > 0$ for normal goods, $\partial x^*_2/\partial M < 0$ for inferior goods
• $h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$

• How does the Hicksian demand change if price $p_i$ changes?

$$\frac{dh_i}{dp_i} = \frac{\partial x_i^*(p, e)}{\partial p_i} + \frac{\partial x_i^*(p, e)}{\partial M} \frac{\partial e(p, \bar{u})}{\partial p_i}$$

• What is $\frac{\partial e(p, \bar{u})}{\partial p_i}$? Envelope theorem:

$$\frac{\partial e(p, \bar{u})}{\partial p_i} = \frac{\partial}{\partial p_i} \left[ p_1 h_1^* + p_2 h_2^* - \lambda (u(h_1^*, h_2^*, \bar{u}) - \bar{u}) \right] = h_i^*(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p, \bar{u}))$$
• Therefore

\[
\frac{\partial h_i (\mathbf{p}, \bar{u})}{\partial p_i} = \frac{\partial x_i^* (\mathbf{p}, e)}{\partial p_i} + \frac{\partial x_i^* (\mathbf{p}, e)}{\partial M} x_i^*(p_1, p_2, e)
\]

• Rewrite as

\[
\frac{\partial x_i^* (\mathbf{p}, M)}{\partial p_i} = \frac{\partial h_i (\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} - x_i^*(p_1, p_2, M) \frac{\partial x_i^* (\mathbf{p}, M)}{\partial M}
\]

• Important result! Allows decomposition into substitution and income effect
• Two effects of change in price:

1. Substitution effect negative: \( \frac{\partial h_i(p, v(p, M))}{\partial p_i} < 0 \)

2. Income effect: \(-x_i^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M}\)

   – negative if good \( i \) is normal \((\frac{\partial x_i^*(p, M)}{\partial M} > 0)\)

   – positive if good \( i \) is inferior \((\frac{\partial x_i^*(p, M)}{\partial M} < 0)\)

• Overall, sign of \( \frac{\partial x_i^*(p, M)}{\partial p_i} \)?

   – negative if good \( i \) is normal

   – it depends if good \( i \) is inferior
• Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

• $x_i^* = \alpha M/p_i$

• $h_i^* =

• Derivative of Hicksian demand with respect to price:

$$\frac{\partial h_i (p, u)}{\partial p_i} =$$

• Rewrite $h_i^*$ as function of $m$: $h_i (p, v(p, M))$

• Compute $v(p, M) =$
• Substitution effect:

\[
\frac{\partial h_i (p, v(p, M))}{\partial p_i} =
\]

• Income effect:

\[
-x_i^* (p_1, p_2, M) \frac{\partial x_i^* (p, M)}{\partial M} =
\]

• Sum them up to get

\[
\frac{\partial x_i^* (p, M)}{\partial p_i} =
\]

• It works!
2 Complements and substitutes

- Nicholson, Ch. 6, pp. 187-192

- How about if price of another good changes?

- Generalize Slutsky equation

- Slutsky Equation:

\[
\frac{\partial x^*_i (p, M)}{\partial p_j} = \frac{\partial h_i (p, v(p, M))}{\partial p_j} - x^*_j(p_1, p_2, M) \frac{\partial x^*_i (p, M)}{\partial M}
\]
• Substitution effect

\[ \frac{\partial h_i(p, v(p, M))}{\partial p_j} > 0 \]

for \( n = 2 \) (two goods). Ambiguous for \( n > 2 \).

• Income effect:

\[ -x_i^*(p_1, p_2, M) \frac{\partial x_i^*(p, M)}{\partial M} \]

– negative if good \( i \) is normal

– positive if good \( i \) is inferior

• How do we define complements and substitutes?
• Def. 1. Goods \( i \) and \( j \) are **gross substitutes** at price \( p \) and income \( M \) if

\[
\frac{\partial x^*_i(p, M)}{\partial p_j} > 0
\]

• Def. 2. Goods \( i \) and \( j \) are **gross complements** at price \( p \) and income \( M \) if

\[
\frac{\partial x^*_i(p, M)}{\partial p_j} < 0
\]

• Example 1 (ctd.): \( x^*_1 = \alpha M/p_1 \), \( x^*_2 = \beta M/p_2 \).

• Gross complements or gross substitutes? Neither!

• Notice: \( \frac{\partial x^*_i(p, M)}{\partial p_j} \) is usually different from \( \frac{\partial x^*_j(p, M)}{\partial p_i} \)
• Def. 3. Goods $i$ and $j$ are **net substitutes** at price $p$ and income $M$ if

$$
\frac{\partial h_i^*(p, v(p, M))}{\partial p_j} = \frac{\partial h_j^*(p, v(p, M))}{\partial p_i} > 0
$$

• Def. 4. Goods $i$ and $j$ are **net complements** at price $p$ and income $M$ if

$$
\frac{\partial h_i^*(p, v(p, M))}{\partial p_j} = \frac{\partial h_j^*(p, v(p, M))}{\partial p_i} < 0
$$

• Example 1 (ctd.): $h_1^* = \bar{u} \left( \frac{\alpha p_2}{1 - \alpha p_1} \right)^{1-\alpha}$

• Net complements or net substitutes? Net substitutes!
3 Labor Supply

• Nicholson Ch. 16, pp. 581-589

• Labor supply decision: how much to work in a day.

• Goods: consumption good $c$, hours worked $h$

• Price of good $p$, hourly wage $w$

• Consumer spends $24 - h = l$ hours in units of leisure

• Utilify function: $u(c, l)$
• Budget constraint?

• Income of consumer: \( M + wh = M + w(24 - l) \)

• Budget constraint: \( pc \leq M + w(24 - l) \) or \( pc + wl \leq M + 24w \)

• Notice: leisure \( l \) is a consumption good with price \( w \). Why?

• General category: opportunity cost

• Instead of enjoying one hour of TV, I could have worked one hour and gained wage \( w \).

• You should value the marginal hour of TV \( w \)! 
• Opportunity costs are very important!

• Example 2. CostCo has a warehouse in SoMa

• SoMa used to have low cost land, adequate for warehouses

• Price of land in SoMa triples in 10 years.

• Should firm relocate the warehouse?
• Did costs of staying in SoMa go up?

• No.

• Did the opportunity cost of staying in SoMa go up?

• Yes!

• Firm can sell at high price and purchase land in cheaper area.
Let’s go back to labor supply.

Maximization problem is
\[
\max u(c, l) \\
\text{s.t. } pc + wl \leq M + 24w
\]

Standard problem (except for 24w)

First order conditions

Assume utility function Cobb-Douglas:
\[
u(c, l) = c^\alpha l^{1-\alpha}\]
- Solution is

\[ c^* = \alpha \frac{M + 24w}{p} \]

\[ l^* = (1 - \alpha) \left( 24 + \frac{M}{w} \right) \]

- Both \( c \) and \( l \) are normal goods

- Unlike in standard Cobb-Douglas problems, \( c^* \) depends on price of other good \( w \)

- Why? Agents are endowed with \( M \) AND 24 hours of \( l \) in this economy

- Normally, agents are only endowed with \( M \)
4 Next Lectures

- Applications:
  - Intertemporal Choice
  - Economics of Altruism