# Economics 101A
# (Lecture 9)

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February 14, 2017

# Outline

1. Slutsky Equation

2. Complements and substitutes

3. Application 1: Labor Supply

# 1 Slutsky Equation

- Nicholson, Ch. 5, pp. 160-163

- Now: go back to Utility Max. in case where $p_2$ increases to $p_2' > p_2$

- What is $\partial x_2^*/\partial p_2$? Decompose effect:

  1. Substitution effect of an increase in $p_i$

     – $\partial h_2^*/\partial p_2$, that is change in EMIN point as $p_2$ descreases

     – Moving along an indifference curve

     – Certainly $\partial h_2^*/\partial p_2 < 0$

2. Income effect of an increase in $p_i$

   - $\partial x_2^*/\partial M$, increase in consumption of good 2 due to increased income

   - Shift out a budget line

   - $\partial x_2^*/\partial M > 0$ for normal goods, $\partial x_2^*/\partial M < 0$ for inferior goods

- $h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$

- How does the Hicksian demand change if price $p_i$ changes?

$$\frac{dh_i}{dp_i} = \frac{\partial x_i^*(\mathbf{p}, e)}{\partial p_i} + \frac{\partial x_i^*(\mathbf{p}, e)}{\partial M} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i}$$

- What is $\frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i}$? Envelope theorem:

$$\begin{aligned} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i} &= \frac{\partial}{\partial p_i} \left[ p_1 h_1^* + p_2 h_2^* - \lambda(u(h_1^*, h_2^*, \bar{u}) - \bar{u}) \right] \\ &= h_i^*(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p, \bar{u})) \end{aligned}$$

- Therefore

$$\frac{\partial h_i\left(\mathbf{p},\bar{u}\right)}{\partial p_i}=\frac{\partial x_i^*\left(\mathbf{p},e\right)}{\partial p_i}+\frac{\partial x_i^*\left(\mathbf{p},e\right)}{\partial M}x_i^*(p_1,p_2,e)$$

- Rewrite as

$$\begin{aligned}\frac{\partial x_i^*\left(\mathbf{p},M\right)}{\partial p_i} &= \frac{\partial h_i\left(\mathbf{p},v(\mathbf{p},M)\right)}{\partial p_i} \\ &\quad -x_i^*(p_1,p_2,M)\frac{\partial x_i^*\left(\mathbf{p},M\right)}{\partial M}\end{aligned}$$

- Important result! Allows decomposition into substitution and income effect

- Two effects of change in price:

  1. Substitution effect negative: $\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} < 0$

  2. Income effect: $-x_i^*(p_1, p_2, M)\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$

     - negative if good $i$ is normal ($\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} > 0$)

     - positive if good $i$ is inferior ($\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} < 0$)

- Overall, sign of $\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_i}$?

  - negative if good $i$ is normal

  - it depends if good $i$ is inferior

- Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

- $x_i^* = \alpha M / p_i$

- $h_i^* =$

- Derivative of Hicksian demand with respect to price:

$$\frac{\partial h_i(\mathbf{p}, \overline{u})}{\partial p_i} =$$

- Rewrite $h_i^*$ as function of $m$: $h_i(\mathbf{p}, v(\mathbf{p}, M))$

- Compute $v(\mathbf{p}, M) =$

- Substitution effect:

$$\frac{\partial h_i\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_i} =$$

- Income effect:

$$-x_i^*(p_1, p_2, M)\frac{\partial x_i^*\left(\mathbf{p}, M\right)}{\partial M} =$$

- Sum them up to get

$$\frac{\partial x_i^*\left(\mathbf{p}, M\right)}{\partial p_i} =$$

- It works!

# 2 Complements and substitutes

- Nicholson, Ch. 6, pp. 187-192

- How about if price of another good changes?

- Generalize Slutsky equation

- Slutsky Equation:

$$\begin{aligned}\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} &= \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} \\ &\quad - x_j^*(p_1, p_2, M)\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}\end{aligned}$$

- Substitution effect

$$\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} > 0$$

for $n = 2$ (two goods). Ambiguous for $n > 2$.

- Income effect:

$$-x_j^*(p_1, p_2, M)\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$$

  - negative if good $i$ is normal
  - positive if good $i$ is inferior

- How do we define complements and substitutes?

- Def. 1. Goods $i$ and $j$ are **gross substitutes** at price $\mathbf{p}$ and income $M$ if

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} > 0$$

- Def. 2. Goods $i$ and $j$ are **gross complements** at price $\mathbf{p}$ and income $M$ if

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_j} < 0$$

- Example 1 (ctd.): $x_1^* = \alpha M / p_1$, $x_2^* = \beta M / p_2$.

- Gross complements or gross substitutes? Neither!

- Notice: $\frac{\partial x_i^*(\mathbf{p},M)}{\partial p_j}$ is usually different from $\frac{\partial x_j^*(\mathbf{p},M)}{\partial p_i}$

- Def. 3. Goods $i$ and $j$ are **net substitutes** at price $\mathbf{p}$ and income $M$ if

$$\frac{\partial h_i^*\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_j} = \frac{\partial h_j^*\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_i} > 0$$

- Def. 4. Goods $i$ and $j$ are **net complements** at price $\mathbf{p}$ and income $M$ if

$$\frac{\partial h_i^*\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_j} = \frac{\partial h_j^*\left(\mathbf{p}, v(\mathbf{p}, M)\right)}{\partial p_i} < 0$$

- Example 1 (ctd.): $h_1^* = \overline{u}\left(\frac{\alpha}{1-\alpha}\frac{p_2}{p_1}\right)^{1-\alpha}$

- Net complements or net substitutes? Net substitutes!

# 3 Labor Supply

- Nicholson Ch. 16, pp. 581-589

- Labor supply decision: how much to work in a day.

- Goods: consumption good $c$, hours worked $h$

- Price of good $p$, hourly wage $w$

- Consumer spends $24 - h = l$ hours in units of leisure

- Utilify function: $u(c, l)$

- Budget constraint?

- Income of consumer: $M + wh = M + w(24 - l)$

- Budget constraint: $pc \leq M + w(24 - l)$ or

$$pc + wl \leq M + 24w$$

- Notice: leisure $l$ is a consumption good with price $w$. Why?

- General category: **opportunity cost**

- Instead of enjoying one hour of TV, I could have worked one hour and gained wage $w$.

- You should value the marginal hour of TV $w$!

- Opportunity costs are very important!

- Example 2. CostCo has a warehouse in SoMa

- SoMa used to have low cost land, adequate for warehouses

- Price of land in SoMa triples in 10 years.

- Should firm relocate the warehouse?

- Did costs of staying in SoMa go up?

- No.

- Did the opportunity cost of staying in SoMa go up?

- Yes!

- Firm can sell at high price and purchase land in cheaper area.

- Let's go back to labor supply

- Maximization problem is

$$\max u(c,l)$$
$$s.t.\ pc + wl \le M + 24w$$

- Standard problem (except for $24w$)

- First order conditions

- Assume utility function Cobb-Douglas:

$$u(c,l) = c^{\alpha} l^{1-\alpha}$$

- Solution is

$$\begin{aligned} c^* &= \alpha \frac{M + 24w}{p} \\ l^* &= (1-\alpha)\left(24 + \frac{M}{w}\right) \end{aligned}$$

- Both $c$ and $l$ are normal goods

- Unlike in standard Cobb-Douglas problems, $c^*$ depends on price of other good $w$

- Why? Agents are endowed with $M$ AND 24 hours of $l$ in this economy

- Normally, agents are only endowed with $M$

# 4 Next Lectures

- Applications:
  - Intertemporal Choice
  - Economics of Altruism