Econ 101A – Spring 2014 – Midterm 1

Problem 1. Utility maximization. (75 points) In this exercise, we consider a standard utility maximization problem with an unusual (for us) income. The utility function is

$$u(x, y) = \alpha \log(x) + (1 - \alpha) \log(y).$$

This function is well-defined for $x > 0$ and for $y > 0$. From now on, assume $x > 0$ and $y > 0$ unless otherwise stated. The price of good x is p_x and the price of good y is p_y. The unusual part is that consumers' income is given not by a monetary budget M, but by endowments of the goods. To be more precise, the consumer starts off owning $\omega_x > 0$ units of good x and $\omega_y > 0$ units of good y. (These numbers are given, treat them as parameters) The consumer can sell the goods so the initial allocation works as a source of wealth. Given this, the income of the consumer is $p_x\omega_x + p_y\omega_y$.

1. Write down the budget constraint. Notice that the budget constraint is standard other than for the income expression, which is given above (5 points)
2. Just for this point, we pick some specific parameter values. Plot the budget lines assuming $\omega_x = 1$, $\omega_y = 1$. Plot first for the case $p_x = 1$, $p_y = 1$, and then plot the budget line for $p_x = 2$, $p_y = 1$. How does the budget line shift? In a new graph, plot the budget constraint for the same two combinations of prices, but assuming $M = 2$. How does the shift in budget line in the case with endowments ω_x and ω_y compare to the standard case with income M? Provide intuition. (10 points)
3. Let's go back to the general case. The consumer maximizes utility subject to the budget constraint with endowments as in point (1). Write down the maximization problem of the consumer with respect to x and y. Explain *briefly* why the budget constraint is satisfied with equality.(5 points)
4. Write down the Lagrangean function. (5 points)
5. Write down the first order conditions for this problem with respect to x, y, and λ. (5 points)
6. Solve explicitly for x^* and y^* as a function of p_x, p_y, ω_x, and ω_y. (5 points)
7. Use the expression for x^* that you obtained in point 6. Differentiate it with respect to p_x, that is, compute $\partial x^*/\partial p_x$. Is this good a Giffen good? (5 points)
8. We are now interested in the sign of $\partial x^*/\partial p_y$. That is, we would like to know if how the demand for one good depends on the price of the other good. Based on this, are these goods (gross) substitutes or complements? (5 points)
9. Consider now how the demand for x^* varies as the endowments ω_x and ω_y vary. That is, compute $\partial x^*/\partial \omega_x$ and $\partial x^*/\partial \omega_y$. Provide an interpretation of these results. (5 points)
10. So far you derived the optimal demands x^* and y^*. Now, consider the demands net of the endowments, that is $z_x^* = x^* - \omega_x$ and $z_y^* = y^* - \omega_y$. These net demands represent the *net demand* for a good. If the net demand is positive, it means that the consumer buys additional quantities relative to the endowment. If the net demand is negative, it indicates that the consumer sells (at least partially) the endowment. Derive expressions for z_x^* and z_y^*. (5 points)
11. Compute how the net demand for good x, that is, z_x^*, varies as the endowment for x varies, that is, compute $\partial z_x^*/\partial \omega_x$. Compare to your answer in point (9) regarding $\partial x^*/\partial \omega_x$ and interpret the difference. (10 points)
12. Suppose now that you have to solve the same problem as above, that is, same budget constraint as in point (1), but the utility function is $v(x, y) = x^a y^{1-\alpha}$. Can you derive the solution for this problem without redoing all the algebra? Explain. (10 point)

Short problem. (Utility Maximization with Graphical Solution) (40 points)

1. Consider an individual with budget constraint $2x + y = 10$. That is, price of x is $p_x = 2$, price of y is $p_y = 1$, and income M equals 10. Plot the budget constraint. (5 points)

2. Plot indifferent curves for the case $u(x, y) = x^2 + y^2$. (Remember: an indifference curve is defined by $u(u, y) - \bar{u} = 0$). (5 points)

3. Using the plots of the indifference curves and the plot of the budget constraint, find graphically the utility-maximizing choice, and solve for x^* and y^*. (Remember that you are maximizing utility subject to the budget constraint and subject to $x \geq 0$ and $y \geq 0$) (10 points)

4. Why in the above case it would not be a good idea to solve the problem with a Lagrangean? (5 points)

5. Now, using the same budget constraint $2x + y = 10$, plot indifference curves for the case $u(x, y) = 4x + y$.(5 points)

6. Using the plots of the indifference curves and the plot of the budget constraint, find graphically the utility-maximizing choice, and solve for x^* and y^*. (5 points)

7. What preferences are the ones in points (2) and (5)? What cases do they indicate? (5 points)