Econ 101A
Midterm 1

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Th 1 October 2009

Do not turn page unless instructed to.

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You have approximately 1 hour and 20 minutes to answer the questions in the midterm. Justin and Mariana will collect the exams at 11.00 sharp. Show your work, and good luck!

Problem 1. Consumption and Leisure Decision: Farmers and Baseball Players. (65 points) Suppose there is an island where there are only 2 types of work available for the islanders: farming and playing baseball. All islanders like playing baseball, while they do not like (per se) farming. The king of the island (randomly) assigns each islander his/her future profession on the day they are born. Islanders are only allowed to work in their assigned profession. This question is about an islander's work decision in their assigned profession when they reach working age.
The utility function for islanders whose profession is farming:

$$u(c, \ell; \alpha, \beta) = c^{\alpha}\ell^{\beta}$$

The utility function for islanders whose profession is playing baseball:

$$u(c, \ell, h; \alpha, \beta, \gamma) = c^{\alpha}\ell^{\beta}h^{\gamma}$$

In this problem we will assume that islanders decide how many hours to work each day, where H is the number of hours in a day (i.e. 24) and h is the number of hours that you work each day. All time not spent working is spent on leisure ℓ. Islanders also get utility from consuming all other goods c and the price of these goods is \$1 (i.e. $p_c = 1$). Assume that $\alpha, \beta, \gamma \in (0, 1)$.

1. Provide intuition of what γ is capturing in the utility function of the baseball players. That is, what does this non-standard consumption-leisure utility function capture? (5 points)

2. First notice that the utility function for islanders whose profession is playing baseball is really a utility function of just 2 endogenous variables (c and ℓ). Rewrite this utility function so that the level of utility is a function of only these choice variables and other exogenous parameters. (5 points)

3. Let's assume that the only income for islanders comes exclusively from working and that to be "fair" the king decides to set the wage that an islander receives for an hour of work to be the same regardless of the type of work. Write the budget constraint that an islander faces in their consumption-leisure decision problem. (5 points)

4. Write down the maximization problem, the Lagrangian, and the first order conditions for the consumption-leisure decision for both the farming islander and the baseball playing islander. (10 points)

5. Solve for c^* and ℓ^* (as well as h^*) for the farming islander. Check that the solution satisfies the constraint $0 \leq l^* \leq H$. (10 points)

6. Still for the farming islander, plot the labor supply function, that is, how the hours of work supplied h^* vary with the wage w. (use h^* on the x axis and w on the y axis). What is the particular feature of this function? Relate to the substitution and income effects. (5 points)

7. Check the second order conditions for the candidate solution you found for farming islanders in the previous part. Use the bordered Hessian. (5 points)

8. The solution for the baseball playing islander is $l^* = (\beta/(\alpha + \beta + \gamma)) * H$. (You are not required to solve for this, though if you do solve for it from the first order condition you get 5 extra credit points). Compare this solution to the solution for l^* for the farmers. Discuss the intuition for how the optimal leisure choices differ. How does l^* vary as γ increases? Discuss the intuition. (5 points)

9. Consider now a special type of baseball players, the workaholic ones. These players do not enjoy leisure time l, while they enjoy working h:

$$u(c, \ell, h; \alpha, \beta, \gamma) = c^{\alpha}h^{\gamma}.$$

Solve for the optimal c^* and l^* in this case. Do not rely on the answer to Question 8. [Hint: Do you need to set up the full Lagrangean to solve for this?] (10 points)

10. The king realizes over time that farmers are unhappy being stuck with their profession. The island needs to have farmers, so the king decides to pay all farmers a transfer of M dollars in addition to their wages. Without solving numerically explain how the king's economic advisor could determine the level of M that each farmer would need to make farmers as happy as (non-workaholic) baseball players. (5 points)

Problem 2. (30 points) **Shorter Problems.**

1. Consider a 2-period economy, $t = 0$ and $t = 1$, as considered in class. In each period, the consumer receives income M_t and decides to consume c_t, with $t = 0, 1$. The prices of goods c_t is equal to 1 in all periods. Write down the intertemporal budget constraint that we use in the utility maximization for the consumption-savings case. (Hint: Start from the last period) (5 points)

2. Now let's generalize that to a 3-period economy, $t = 0$, $t = 1$, and $t = 2$. In each period, the consumer receives income M_t and decides to consume c_t, with $t = 0, 1, 2$. The prices of goods c_t is still equal to 1 in all periods. Write down the intertemporal budget constraint that we use in the utility maximization for the consumption-savings case. (Again: Start from the last period) Can you conjecture how the constraint looks like for a t-period economy? (10 points)

3. Consider a version of the classical Condorcet paradox. Suppose we have three political candidates, A, B, and C, and that there are three voters with preferences as follows (candidates being listed in *decreasing* order of preference):

   - Voter 1: A B C
   - Voter 2: B C A
   - Voter 3: C A B

Now define societal preferences over $\{A, B, C\}$ as follows: $x \succsim y$ if at least two voters prefer x to y. So, for example, $A \succsim B$ if at least two voters prefer A over B. Using this definition of the weak preference relation $\succsim$, prove (or show that it is false) that these preferences are: (i) complete; (ii) transitive. Be clear and complete in your claims. Can you provide a utility function that represents these preferences? (15 points)