Problem 1. Three-Good Cobb-Douglas. (50 points) Seung likes three goods: $x_1$, $x_2$, and $x_3$. He is aware that in Econ 101A we only use two goods, but he is too attached to all of them to let go of one. He maximizes the utility function

$$u(x_1, x_2, x_3) = x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3},$$

with $0 < \alpha_i < 1$ for $i = 1, 2, 3$. The consumption good $x_i$ has price $p_i$ (for $i = 1, 2, 3$) and the individual has total income $M$.

1. Compute the marginal utility of consumption with respect to good $x_1$, $\partial u(x_1, x_2, x_3)/\partial x_1$. (2 points)
2. What is the limit of the marginal utility for $x_1 \to 0$ and for $x_1 \to \infty$? Interpret the economic intuition behind this feature of this utility function. (5 points)
3. Write the budget constraint. (3 points)
4. Write the maximization problem of Seung. Seung wants to achieve the highest utility subject to the budget constraint. Write down the boundary constraints for $x_1, x_2, x_3$, and neglect them for now. (3 points)
5. Assuming that the budget constraint holds with equality, write down the Lagrangean and derive the first order conditions with respect to $x_1, x_2, x_3$, and $\lambda$. (5 points)
6. Solve for $x_1^*$ as a function of the prices $p_1, p_2, p_3$, the income $M$, and the parameters $\alpha_1, \alpha_2$, and $\alpha_3$. [Hint: combine the first and second first-order condition, then combine the first and third first-order condition, and finally plug in budget constraint] Similarly solve for $x_2^*$ and $x_3^*$. (6 points)
7. True or false? Show your work: “Cobb-Douglas preferences have the feature that the share of money spent on each good does not depend on the income, or on prices” (6 points)
8. Are the boundary conditions for $x_1, x_2$, and $x_3$ satisfied? (2 points)
9. Is good $x_1$ a normal good (for all values of $M$ and prices $p_i$)? Compute and answer. (4 points)
10. Plot the implied demand function for $x_1$, that is plot $x_1$ as a function of $p_1$. (Put $p_1$ on the y axis and $x_1$ on the x axis) (4 points)
11. Is good $x_1$ a Giffen good? Why did you know this already from the answer to question 9? (5 points)
12. Are goods $x_1$ and $x_2$ gross complements, gross substitutes, or neither? Define and answer. (5 points)

Problem 2. (26 points)

1. Angela has utility function $u(x_1, x_2) = 2x_1 + 2x_2$.
   (a) Plot the indifference curves of Angela. What kind of goods do they represent? (4 points)
   (b) Using the plot you did, find the utility-maximizing solution $x_1^*, x_2^*$ for prices $p_1 = 1, p_2 = 2$ and income $M$. Argue the steps you make. (8 points)

2. Kim has utility function $u(x_1, x_2) = \min(x_1, 2x_2)$
   (a) Plot the indifference curves of Kim. What kind of goods do they represent? (4 points)
   (b) Are the preferences represented by this utility function monotonic? Define. (4 points)
   (c) Are they strictly monotonic? Define. (6 points)