Econ 101A – Midterm 1
Th 28 September 2006.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. Adriana and Suresh will collect the exams at 11.00 sharp. Show your work, and good luck!

Problem 1. Quasi-linear utility for leisure. (55 points) Martha is deciding how many hours of leisure l she should take on a typical day, and how much she should consume. Martha's utility function is $u(c,l) = \alpha c + \phi(l)$, where $\phi(l)$ is a function satisfying $\phi'(l) > 0$ and $\phi''(l) < 0$. Assume $\alpha > 0$. The consumption good c has price 1.

1. Compute the marginal utility of consumption $\partial u(c,l)/\partial c$ and of leisure $\partial u(c,l)/\partial l$. What is the special feature of this utility function? (3 points)
2. Martha maximizes $u(c,l)$ subject to a budget constraint. Martha starts the period with income M, and earns $w > 0$ for each hour h worked. Every hour of work is subtracted from leisure, so $l = 24 - h$. Write down the budget constraint as a function of c and l. Justify the steps. (4 points)
3. Write down the maximization problem of Martha that wants to achieve the highest utility subject to the budget constraint. Martha maximizes with respect to c and l. Write down the boundary constraints for c and l, and neglect them from now on. (3 points)
4. Assuming that the budget constraint holds with equality, write down the Lagrangean and derive the first order conditions with respect to c, l, and λ. (2 points)
5. Solve for λ^* as a function of the parameters, α, M, w. What does λ^* depend on? (3 points)
6. Use the envelope function for constrained maximization to show that λ^* equals $\partial v(\alpha, M, w)/\partial M$, that is, λ represents the marginal utility of wealth. Remember, $v(\alpha, M, w)$ is the indirect utility function, that is, $v(\alpha, M, w) = u(c^*(\alpha, M, w), l^*(\alpha, M, w))$. [Note: You will not need to explicitly solve for c^* and l^* to do this] (6 points)
7. Combine 5 and 6 to solve for $\partial v(\alpha, M, w)/\partial M$. Why is this the case? Relate this to your answer in point 1, providing as much intuition as you can. (4 points)
8. Going back to the maximization problem, plug the value of λ^* into the first order condition for l. Use the condition you obtain to derive the comparative statics of leisure with respect to income M ($\partial l^*(\alpha, M, w)/\partial M$) and wage ($\partial l^*(\alpha, M, w)/\partial w$). What is the sign of these derivatives? [You will need the implicit function theorem for at least one of these] (5 points)
9. Given that M and w are both sources of earnings, why is the effect of changes on M and w on l^* so different? Provide as detailed an answer as you can. Give economic intuition. (7 points)
10. Now provide an equation that denotes the solution for $c^*(\alpha, M, w)$. [It may be helpful to define $\omega(.) = (\phi')^{-1}(.)$ as the inverse function of $\phi'(.)$]. Could you run into problems with the non-negativity constraints for c^* and l^*? Can you give an example of values of α, M, and w such that you do violate non-negativity? (5 points)
11. (Harder) What is the solution if the non-negativity constraints are violated? (6 points)
12. We forgot the second order conditions! Compute the Bordered Hessian and check that the sufficient conditions for an optimum are satisfied. (7 points)

Problem 2. Preferences. (25 points)

1. Consider a preference relation $\succsim$ with the properties of completeness and transitivity. Define what we mean by completeness and transitivity of a relation. (3 points)

2. As we discussed in class, a preference relation $\succsim$ defines the indifference relation $\sim$ as follows: $x \sim y$ if and only if $x \succsim y$ and $y \succsim x$. Here comes the question: If $\succsim$ is complete and transitive, does this imply that the relation $\sim$ is complete? Provide a proof or, if the statement is false, an example to the contrary (6 points)

3. If $\succsim$ is complete and transitive, does it imply that $\sim$ is transitive? Provide a proof or, if the statement is false, an example to the contrary (3 points)

4. Andrew is religious and believe in meditation. He is detached from material things, but values prayer and meditation highly. As he states his preferences,"I like meditation, I would always rather always do more of it, and am completely indifferent as to the consumption of material goods". Denote by m the number of hours of mediations, and by c the quantity of material good consumed. We can translate these preferences as follows. When comparing two bundles x and y, with $x = (m_x, c_x)$ and $y = (m_y, c_y)$, Andrew's preferences are such that $x \succsim y$ if and only if $m_x \geq m_y$. Provide the intuition for why this is the case and plot indifference curves in the two-dimensional space (m, c). (3 points)

5. Are these preferences monotonic? Are they strictly monotonic? Argue the answer, and define the terms used. (4 points)

6. Are these preferences convex? Are they continuous? Argue the answer, and define the terms used. (6 points)