Econ 101A – Midterm 1
Tu 11 October 2005.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. Adriana and Vikram will collect the exams at 12.30 sharp. Show your work, and good luck!

Problem 1. Utility maximization and charitable giving. (49 points) Consider Mario, a Californian resident with income M. Mario cares about own consumption c, and about a charitable project G in Louisiana. The welfare effects of the project for Louisiana citizens are measured by $h(g+S)$, where g is Mario's charitable donation and S is the seed money donated by others. Mario derives utility $\alpha h(g+S)$ from the project, with $\alpha \geq 0$. Therefore, Mario maximizes

$$\begin{aligned} &\max_{c,g} u(c) + \alpha h(g+S) \\ &s.t.\ c+g \leq M \\ &s.t. c \geq 0 \\ &s.t. g \geq 0 \end{aligned} \tag{1}$$

We assume $u'() > 0$, $u''() < 0$, $h'() > 0$ and $h''() < 0$. That is, both u and h are increasing, concave functions. The function h is concave because there are diminishing returns to the charitable project.

1. What is the interpretation of α in the utility function? Interpret in particular the special case $\alpha = 0$. (3 points)
2. Argue that the problem can be rewritten as
$$\begin{aligned} &\max_{g} u(M-g) + \alpha h(g+S) \\ &s.t. 0 \leq g \leq M. \end{aligned}$$
(4 points)
3. Neglecting the constraint $0 \leq g \leq M$ for now, derive first order and second order conditions. Argue that the point identified by the first-order condition is a maximum. (6 points)
4. Solve for g^* and c^* for the case $u(c) = \log(c)$ and $h(G) = \log(G)$ using the first-order conditions. (4 points)
5. Keep assuming $u(c) = \log(c)$ and $h(G) = \log(G)$. Now it is time to check the constraint $0 \leq g \leq M$. Is the constraint $g \leq M$ always satisfied? Is the constraint $g \geq 0$ always satisfied? If not, provide an example to the converse. Write the solution for g^* and c^*, taking into account corner solutions (8 points)
6. Keep assuming $u(c) = \log(c)$ and $h(G) = \log(G)$. How do c^* and g^* depend on M, α, and S? Provide intuition for each of these comparative statics, in particular for the latter. (6 points)
7. Now we go back to the general formulation with $u()$ and $h()$ concave. Using the implicit function theorem, and neglecting corner solutions, show that $\partial g^*/\partial S < 0$. Comment on this result. (6 points)
8. (Open-ended) An economist, John List, does an empirical study of the effect of seed money (the money collected early on in a fundraising drive) on charitable donations. He finds that higher seed money is associated with *higher* donations, against the prediction in point 7. What can be an explanation of this finding? (6 points)
9. Keep assuming $u()$ and $h()$ concave. Consider the indirect utility $V(S,M,\alpha) = u(M - g^*(S,M,\alpha)) + \alpha h(g^*(S,M,\alpha) + S)$. Use the envelope theorem to compute $dV(S,M,\alpha)/dS$, that is, how the indirect utility vary as the seed money increases. Under what conditions is $\partial V(S,M,\alpha)/\partial S > 0$ and why? (6 points)

Problem 2. (24 points)

1. You are a consultant for the pumpkin industry. The producers of pumpkins foresee a (small) income increase for the consumers of pumpkins and consult you to predict how this will affect the demand for pumpkins. You observed that, as pumpkin price increases, demand for pumpkin increases. Provide a formal answer to the pumpkin producers and provide intuition. (8 points)

2. Jenny faces a choice set $X = \{A, B, C, D\}$. Her preferences are defined by $A \succsim B, B \succsim D, B \succsim C$. Are these preferences complete? Are these preferences transitive? (8 points)

3. McDo faces a choice set $X = \{fries, apple, sundae\}$. McDo prefers fries to apples, because fries are more appetizing ($fries \succ apple$). McDo prefers apples to sundaes, because a sundae is exceedingly caloric. ($apple \succ sundae$). Finally, McDo prefers sundaes to fries because, once the comparison is between fatty foods, she prefers the sweet one ($sundae \succ fries$). These relations define McDo's preference. Can you represent these preferences with a utility function? (8 points)