

# Economics 101A

## (Lecture 8)

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## Outline

1. Comparative Statics (Introduction)
2. Income Changes
3. Price Changes
4. Expenditure Minimization
5. Slutsky Equation

# 1 Comparative Statics (introduction)

- Nicholson, Ch. 5, pp. 145-155
- Utility maximization yields  $x_i^* = x_i^*(p_1, p_2, M)$
- Quantity consumed as a function of income and price
  
- What happens to quantity consumed  $x_i^*$  as prices or income varies?

- Simple case: Equal increase in prices and income.

- $M' = tM, p'_1 = tp_1, p'_2 = tp_2.$

- Compare  $x^*(tM, tp_1, tp_2)$  and  $x^*(M, p_1, p_2).$

- What happens?

- Write budget line:  $tp_1x_1 + tp_2x_2 = tM$

- Demand is homogeneous of degree 0 in  $\mathbf{p}$  and  $M$ :

$$x^*(tM, tp_1, tp_2) = t^0 x^*(M, p_1, p_2) = x^*(M, p_1, p_2).$$

- Consider Cobb-Douglas Case:

$$x_1^* = \frac{\alpha}{\alpha + \beta} M/p_1, x_2^* = \frac{\beta}{\alpha + \beta} M/p_2$$

- What is  $\partial x_1^*/\partial M$ ?

- What is  $\partial x_1^*/\partial p_1$ ?

- What is  $\partial x_1^*/\partial p_2$ ?

- General results?

## 2 Income changes

- Income increases from  $M$  to  $M' > M$ .
- Budget line ( $p_1x_1 + p_2x_2 = M$ ) shifts out:

$$x_2 = \frac{M'}{p_2} - x_1 \frac{p_1}{p_2}$$

- New optimum?

- Engel curve:  $x_i^*(M)$  : demand for good  $i$  as function of income  $M$  holding fixed prices  $p_1, p_2$

- Does  $x_i^*$  increase with  $M$ ?

- Yes. Good  $i$  is *normal*

- No. Good  $i$  is *inferior*

### 3 Price changes

- Price of good  $i$  decreases from  $p_i$  to  $p'_i > p_i$
- For example, decrease in price of good 2,  $p'_2 < p_2$
- Budget line tilts:

$$x_2 = \frac{M}{p'_2} - x_1 \frac{p_1}{p'_2}$$

- New optimum?



- Does  $x_i^*$  decrease with  $p_i$ ?

- Yes. Most cases

- No. Good  $i$  is *Giffen*

- Ex.: Potatoes in Ireland

- Do not confuse with Veblen effect for luxury goods or informational asymmetries: these effects are real, but not included in current model

## 4 Expenditure minimization

- Nicholson, Ch. 4, pp. 131-135; Ch. 5, pp. 155-157
- Solve problem **EMIN** (minimize expenditure):

$$\begin{aligned} \min p_1x_1 + p_2x_2 \\ \text{s.t. } u(x_1, x_2) \geq \bar{u} \end{aligned}$$

- Choose bundle that attains utility  $\bar{u}$  with minimal expenditure
- Ex.: You are choosing combination CDs/restaurant to make a friend happy
- If utility  $u$  strictly increasing in  $x_i$ , can maximize s.t. equality
- Denote by  $h_i(p_1, p_2, \bar{u})$  solution to EMIN problem
- $h_i(p_1, p_2, \bar{u})$  is *Hicksian or compensated demand*

- Graphically:
  - Fix indifference curve at level  $\bar{u}$
  - Consider budget sets with different  $M$
  - Pick budget set which is tangent to indifference curve

- Optimum coincides with optimum of Utility Maximization!

- Formally:

$$h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$$

- Expenditure function is expenditure at optimum
- $e(p_1, p_2, \bar{u}) = p_1 h_1(p_1, p_2, \bar{u}) + p_2 h_2(p_1, p_2, \bar{u})$
- $h_i(p_i)$  is *Hicksian or compensated demand* function
- Is  $h_i$  always decreasing in  $p_i$ ? Yes!
- Graphical proof: moving along a convex indifference curve
- (For non-convex indifferent curves, still true)

- Using first order conditions:

$$L(x_1, x_2, \lambda) = p_1x_1 + p_2x_2 - \lambda(u(x_1, x_2) - \bar{u})$$

$$\frac{\partial L}{\partial x_i} = p_i - \lambda u'_i(x_1, x_2) = 0$$

- Write as ratios:

$$\frac{u'_1(x_1, x_2)}{u'_2(x_1, x_2)} = \frac{p_1}{p_2}$$

- $MRS$  = ratio of prices as in utility maximization!
- However: different constraint  $\implies \lambda$  is different

- Example 1: Cobb-Douglas utility

$$\begin{aligned} \min & p_1 x_1 + p_2 x_2 \\ \text{s.t.} & x_1^\alpha x_2^{1-\alpha} \geq \bar{u} \end{aligned}$$

- Lagrangean =

- F.o.c.:

- Solution:  $h_1^* =$  ,  $h_2^* =$

- $\partial h_i^* / \partial p_i < 0$ ,  $\partial h_i^* / \partial p_j > 0$ ,  $j \neq i$

# 5 Slutsky Equation

- Nicholson, Ch. 5, pp. 160-163
- Now: go back to Utility Max. in case where  $p_2$  increases to  $p'_2 > p_2$
- What is  $\partial x_2^*/\partial p_2$ ? Decompose effect:
  1. Substitution effect of an increase in  $p_i$ 
    - $\partial h_2^*/\partial p_2$ , that is change in EMIN point as  $p_2$  decreases
    - Moving along an indifference curve
    - Certainly  $\partial h_2^*/\partial p_2 < 0$

2. Income effect of an increase in  $p_i$

- $\partial x_2^*/\partial M$ , increase in consumption of good 2 due to increased income
- Shift out a budget line
- $\partial x_2^*/\partial M > 0$  for normal goods,  $\partial x_2^*/\partial M < 0$  for inferior goods

- $h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$
- How does the Hicksian demand change if price  $p_i$  changes?

$$\frac{dh_i}{dp_i} = \frac{\partial x_i^*(\mathbf{p}, e)}{\partial p_i} + \frac{\partial x_i^*(\mathbf{p}, e)}{\partial M} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i}$$

- What is  $\frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i}$ ? Envelope theorem:

$$\begin{aligned} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i} &= \frac{\partial}{\partial p_i} [p_1 h_1^* + p_2 h_2^* - \lambda(u(h_1^*, h_2^*, \bar{u}) - \bar{u})] \\ &= h_i^*(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p, \bar{u})) \end{aligned}$$

- Therefore

$$\frac{\partial h_i(\mathbf{p}, \bar{u})}{\partial p_i} = \frac{\partial x_i^*(\mathbf{p}, e)}{\partial p_i} + \frac{\partial x_i^*(\mathbf{p}, e)}{\partial M} x_1^*(p_1, p_2, e)$$

- Rewrite as

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_i} = \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} - x_i^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$$

- Important result! Allows decomposition into substitution and income effect

- Two effects of change in price:

1. Substitution effect negative:  $\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} < 0$

2. Income effect:  $-x_i^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$

- negative if good  $i$  is normal  $(\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} > 0)$

- positive if good  $i$  is inferior  $(\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} < 0)$

- Overall, sign of  $\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_i}$ ?

- negative if good  $i$  is normal

- it depends if good  $i$  is inferior

- Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

- $x_i^* = \alpha M/p_i$

- $h_i^* =$

- Derivative of Hicksian demand with respect to price:

$$\frac{\partial h_i(\mathbf{p}, \bar{u})}{\partial p_i} =$$

- Rewrite  $h_i^*$  as function of  $m$ :  $h_i(\mathbf{p}, v(\mathbf{p}, M))$

- Compute  $v(\mathbf{p}, M) =$

- Substitution effect:

$$\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} =$$

- Income effect:

$$-x_i^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} =$$

- Sum them up to get

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_i} =$$

- It works!

## 6 Next Lectures

- Complements and Substitutes
- Then moving on to applications:
  - Labor Supply
  - Intertemporal choice
  - Economics of Altruism