Outline

1. Who are we?

2. Prerequisites for the course

3. A test in math

4. The economics of discrimination

5. Optimization with 1 variable
1 Who are we?

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- Professor of Economics

- Bocconi (Italy) undergraduate (Econ.), Harvard PhD (Econ.)

- Psychology and Economics (Behavioral Economics), Economics of Media

- Evans 515, Office Hours: Th 11.30-1.30
Grant Graziani (2 Sections)

- Graduate Student, Department of Economics
- Office Hours: M 4-6PM, 636 Evans

Peter Jones (2 Sections)

- Graduate Student, Department of Economics
- Office Hours: Tu 11AM-1PM, 636 Evans
2 Prerequisites

- Mathematics
  - Good knowledge of multivariate calculus – Math 1A or 1B and 53
  - Basic knowledge of probability theory and matrix algebra

- Economics
  - Knowledge of fundamentals – Ec1 or 2 or 3
  - High interest!
• Go over syllabus
3 A Test in Math

1. Can you differentiate the following functions with respect to $x$?

(a) $y = \exp(x)$

(b) $y = a + bx + cx^2$

(c) $y = \frac{\exp(x)}{b^x}$

2. Can you partially differentiate these functions with respect to $x$ and $w$?

(a) $y = axw + bx - c \frac{x}{w} + d\sqrt{xw}$

(b) $y = \exp(x/w)$

(c) $y = \int_0^1 (x + aw^2 + xs)ds$
3. Can you plot the following functions of one variable?

(a) $y = \exp(x)$

(b) $y = -x^2$

(c) $y = \exp(-x^2)$

4. Are the following functions concave, convex or neither?

(a) $y = x^3$

(b) $y = -\exp(x)$

(c) $y = x^5 y^5$ for $x > 0, y > 0$
5. Consider an urn with 20 red and 40 black balls?

(a) What is the probability of drawing a red ball?

(b) What is the probability of drawing a black ball?

6. What is the determinant of the following matrices?

(a) \[ A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \]

(b) \[ A = \begin{bmatrix} 10 & 10 \\ 20 & 20 \end{bmatrix} \]
4 The economics of discrimination

- What do economists know about discrimination?
  - Model of discrimination in workplace (inspired by Becker, *Economics of Discrimination*, 1957)

- Workers:
  - A and B. They produce 1 widget per day
  - Both have reservation wage $\bar{u}$

- Firm:
  - Sells widgets at price $p > \bar{u}$ (assume $p$ given)
  - Dislikes worker B
  - Maximizes profits ($p^* \text{ no of widgets} - \text{cost of labor}) - \text{disutility} d$ if employs B
• Wages and employment in this industry?

• Employment

  – Net surplus from employing $A$: $p - \bar{u}$

  – Net surplus from employing $B$: $p - \bar{u} - d$

  – If $\bar{u} < p < \bar{u} + d$, Firm employs $A$ but not $B$

  – If $\bar{u} + d < p$, Firm employs both

• What about wages?
• Case I. Firm monopolist/monopsonist and no union
  – Firm maximizes profits and gets all the net surplus
  – Wages of $A$ and $B$ equal $\bar{u}$

• Case II. Firm monopolist/monopsonist and union
  – Firm and worker get half of the net surplus each
  – Wage of $A$ equals $\bar{u} + 0.5 \times (p - \bar{u})$
  – Wage of $B$ equals $\bar{u} + 0.5 \times (p - \bar{u} - d)$

• Case III. Perfect competition among firms that discriminate ($d > 0$)
  – Prices are lowered to the cost of production
  – Wage of $A$ equals $p \ (= \bar{u})$
  – $B$ is not employed
• The magic of competition

• Case IIIb. Perfect competition + At least one firm does not discriminate \((d = 0)\)
  
  – This firm offers wage \(p\) to both workers
  
  – What happens to worker \(B\)?
  
  – She goes to the firm with \(d = 0\)!
  
  – In equilibrium now:
    
    * Wage of \(A\) equals \(p\)
    
    * Wage of \(B\) equals \(p\) as well!

• Competition eliminates the pay and employment differential between men and women
• Is this true? Any evidence?

  
  – Local monopolies in banking industry until mid 70s
  
  – Mid 70s: deregulation
  
  – From local monopolies to perfect competition.
  
  – Wages?
    * Wages fall by 6.1 percent
  
  – Discrimination?
    * Wages fall by 12.5 percent for men
    * Wages fall by 2.9 percent for women
    * Employment of women as managers increases by 10 percent
Summary: Competition is not great for workers (wages go down)

BUT: Drives away the gender gap
• More evidence on discrimination: Does black-white and male-female wage back derive from discrimination?

• Field experiment (Bertrand and Mullainathan, *American Economic Review*, 2004)

• Send real CV with randomly picked names:
  
  – Male/Female
  
  – White/African American

<table>
<thead>
<tr>
<th>White Female</th>
<th>Perception</th>
<th>African American Female</th>
<th>Perception</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>$\frac{L(W)}{L(B)}$</td>
<td>Name</td>
<td>$\frac{L(B)}{L(W)}$</td>
</tr>
<tr>
<td>Allison</td>
<td>$\infty$</td>
<td>0.926</td>
<td></td>
</tr>
<tr>
<td>Anne</td>
<td>$\infty$</td>
<td>0.962</td>
<td></td>
</tr>
<tr>
<td>Carrie</td>
<td>$\infty$</td>
<td>0.923</td>
<td></td>
</tr>
<tr>
<td>Emily</td>
<td>$\infty$</td>
<td>0.925</td>
<td></td>
</tr>
<tr>
<td>Jill</td>
<td>$\infty$</td>
<td>0.889</td>
<td></td>
</tr>
<tr>
<td>Laurie</td>
<td>$\infty$</td>
<td>0.963</td>
<td></td>
</tr>
<tr>
<td>Kristen</td>
<td>$\infty$</td>
<td>0.963</td>
<td></td>
</tr>
<tr>
<td>Meredith</td>
<td>$\infty$</td>
<td>0.926</td>
<td></td>
</tr>
<tr>
<td>Sarah</td>
<td>$\infty$</td>
<td>0.852</td>
<td></td>
</tr>
</tbody>
</table>
• Measure call-back rate from interview

– Results (Table 1):

  * Call-back rates 50 percent higher for Whites!
  * No effect for Male-Female call back rates

<table>
<thead>
<tr>
<th>Sample:</th>
<th>Callback Rate for White Names</th>
<th>Callback Rate for African American Names</th>
<th>Ratio</th>
<th>Difference (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All sent resumes</td>
<td>9.65% ([2435])</td>
<td>6.45% ([2435])</td>
<td>1.50</td>
<td>3.20% (0.0000)</td>
</tr>
<tr>
<td>Chicago</td>
<td>8.06% ([1352])</td>
<td>5.40% ([1352])</td>
<td>1.49</td>
<td>2.66% (0.0057)</td>
</tr>
<tr>
<td>Boston</td>
<td>11.63% ([1083])</td>
<td>7.76% ([1083])</td>
<td>1.50</td>
<td>4.05% (0.0023)</td>
</tr>
<tr>
<td>Females</td>
<td>9.89% ([1860])</td>
<td>6.63% ([1886])</td>
<td>1.49</td>
<td>3.26% (0.0003)</td>
</tr>
<tr>
<td>Females in administrative jobs</td>
<td>10.46% ([1358])</td>
<td>6.55% ([1359])</td>
<td>1.60</td>
<td>3.91% (0.0003)</td>
</tr>
<tr>
<td>Females in sales jobs</td>
<td>8.37% ([502])</td>
<td>6.83% ([527])</td>
<td>1.22</td>
<td>1.54% (0.3523)</td>
</tr>
<tr>
<td>Males</td>
<td>8.87% ([575])</td>
<td>5.83% ([549])</td>
<td>1.52</td>
<td>3.04% (0.0513)</td>
</tr>
</tbody>
</table>

*Notes:
1. The table reports, for the entire sample and different subsamples of sent resumes, the callback rates for applicants with a White sounding name (column 1) and an African American sounding name (column 2), as well as the ratio (column 3) and difference (column 4) of these callback rates. In brackets in each cell is the number of resumes sent in that cell.
2. Column 4 also reports the p-value for a test of proportion testing the null hypothesis that the callback rates are equal across racial groups.
• Strong evidence of discrimination against African Americans

  – Did Obama change this? Interesting question

• Example of Applied Microeconomics

  – Not covered in this class: See Ec140-141-142 (Econometrics and Applied Metrics) and 131 (Public), 157 (Health), and 172 (Development)

  – Also: URAP – Get involved in a professor’s research

  – If curious: read Steven Levitt and Stephen Dubner, *Freakonomics*

• At end of semester, more examples
5 Optimization with 1 variable

- Nicholson, Ch.2, pp. 20-23

- Example. Function \( y = -x^2 \)

- What is the maximum?

- Maximum is at 0

- General method?
• Sure! Use derivatives

• Derivative is slope of the function at a point:

\[
\frac{\partial f(x)}{\partial x} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

• Necessary condition for maximum \( x^* \) is

\[
\frac{\partial f(x^*)}{\partial x} = 0 \quad (1)
\]

• Try with \( y = -x^2 \).

\[
\frac{\partial f(x)}{\partial x} = 0 \implies x^* =
\]
• Does this guarantee a maximum? No!

• Consider the function $y = x^3$

  \[
  \frac{\partial f(x)}{\partial x} = \quad = 0 \quad \Rightarrow \quad x^* =
  \]

• Plot $y = x^3$. 

• Sufficient condition for a (local) maximum:

\[ \frac{\partial f(x^*)}{\partial x} = 0 \text{ and } \left. \frac{\partial^2 f(x)}{\partial^2 x} \right|_{x^*} < 0 \quad (2) \]

• Proof: At a maximum, \( f(x^* + h) - f(x^*) < 0 \) for all \( h \).

• Taylor Rule: \( f(x^* + h) - f(x^*) = \frac{\partial f(x^*)}{\partial x} h + \frac{1}{2} \left. \frac{\partial^2 f(x^*)}{\partial^2 x} \right|_{x^*} h^2 + \text{higher order terms.} \)

• Notice: \( \frac{\partial f(x^*)}{\partial x} = 0. \)

• \( f(x^* + h) - f(x^*) < 0 \) for all \( h \) \( \implies \) \( \left. \frac{\partial^2 f(x^*)}{\partial^2 x} \right|_{x^*} h^2 < 0 \)
\( 0 \implies \left. \frac{\partial^2 f(x^*)}{\partial^2 x} \right|_{x^*} < 0 \)

• Careful: Maximum may not exist: \( y = \exp(x) \)
• Tricky examples:

- *Minimum.* \( y = x^2 \)

- *No maximum.* \( y = \exp(x) \) for \( x \in (-\infty, +\infty) \)

- *Corner solution.* \( y = x \) for \( x \in [0, 1] \)
6 Next Class

- Multivariate Maximization
- Comparative Statics
- Implicit Function Theorem
- Envelope Theorem

Going toward:
- Preferences
- Utility Maximization (where we get to apply maximization techniques the first time)