

Problem Set 11
Due in lecture Thursday, December 1

1. Consider deposit insurance in the Diamond-Dybvig model as presented in lecture and in the reading.
 - a. If fraction $\phi > \theta$ of depositors withdraw in period 1, how large a tax must the government levy on each agent in period 1 to be able to increase the total consumption of the nonwithdrawers in the two periods to c_2^{b*} ? Explain why your answer should simplify to zero when $\phi = \theta$, and check that it does.
 - b. Suppose the tax is marginally less than the amount you found in part (a). Would the type b's still prefer to wait until period 2 rather than try to withdraw in period 1?

2. (This follows Jacklin, 1987.) Consider the Diamond-Dybvig model as presented in lecture and in the reading. But suppose that instead of a bank, there is a firm. The firm obtains S units of the economy's endowment by selling S shares in period 0 (the price of a share in units of period 0 endowment is 1). The firm's business plan (to which it is committed) is to invest the S units; pay a dividend of D_1 per share in period 1 (by liquidating fraction D_1 of its investment); and then pay a dividend of D_2 per share in period 2 that leaves it with no remaining assets.
 - a. Explain why $D_2 = R(1 - D_1)$.
 - b. Suppose all agents use their endowment to buy shares in the firm, and suppose there is a market for shares in the firm in period 1 after D_1 has been paid. If all type a agents sell their shares and all type b agents use all of their period 1 dividends to buy shares, what will the price of shares, P , be as a function of D_1 and θ ?
 - c. Continue to assume that all type a agents sell their shares and all type b agents use all of their period 1 dividends to buy shares. What will be the consumption of type a agents in period 1? The consumption of type b agents in period 2?
 - d. Is there a value of D_1 that yields the social optimum? Explain. (Recall that the social optimum is for the type a agents to consume $1/[\theta + (1 - \theta)\rho]$ in period 1 and for the type b's to consume $\rho R/[\theta + (1 - \theta)\rho]$ in period b.)
 - e. For what values of P will the type a agents want to sell their shares (or be indifferent)? For what values of P will the type b agents want to buy shares (or be indifferent)? With D_1 equal to the value you found in part (d), are these conditions satisfied?
 - f. Is this equilibrium vulnerable to a run? Explain.

3. In the Diamond-Dybvig model, the key departure from Walrasian assumptions is:
 - A. Asymmetric information between entrepreneurs and outside investors.
 - B. The presence of noise traders.
 - C. Preference shocks.
 - D. The lack of observability of agents' types.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ANSWERS WILL GENERALLY NOT BE PROVIDED)

4. Consider Problem 6 on Problem Set 10. For simplicity, assume $A_0 = 0$. Now, however, there is a third type of agent: hedge fund managers. They are born in period 0 and care only about consumption in period 2. Like the sophisticated investors, they have utility $U(C) = -e^{-2\gamma C}$ and no initial wealth. There are A_H of them. They participate in the market for the risky asset in period 0, and do not make any additional trades in period 1. However, they face a cost if they incur short-term losses, and gain a reward if they obtain short-term gains. Specifically, if a hedge fund manager purchases amount H of the risky asset, he or she receives $aH(P_1 - E_0[P_1])$ in period 1, where $a > 0$ and where P_1 is the price of the risky asset in period 1. The manager then holds this payment in the safe asset from period 1 to period 2, and so it adds to (or subtracts from) his or her period-2 consumption.

a. Consider first period 1. Find an expression for $P_1 - (1 + F_1)$ taking the period-0 purchases of the hedge fund managers, $X_0^h A_H$, as given. (Hint: In period 1, the demand of the hedge fund managers is fixed and does not respond to the price of the asset. As a result, in period 1 their demand enters in the same way as that of the noise traders.)

b. Now consider period 0.

i. Find an expression for the representative hedge fund manager's period-2 consumption as a function of $P_0, F_1, F_2, a, P_1 - E_0[P_1]$, and X_0^h .

ii. What are the mean and variance of his or her second-period consumption as a function of $P_0, a, V_1^F, V_2^F, \gamma, A_1, V_2^N$, and X_0^h ?

iii. What is the first-order condition for his or her choice of X_0^h ?

iv. Use your results to find an expression for $P_0 - 1$.

v. Does greater agency risk (a higher a) increase the impact of the noise traders in period 0 on the price of the asset?

5. Consider Problem 6 on Problem Set 10. Suppose, however, that the demand of the period-0 noise traders is not fully persistent, so that noise traders' demand in period 1 is $\rho N_0 + N_1, \rho < 1$. How, if at all, does this affect your answer in part b(iii) of Problem 6 on Problem Set 10 for how the noise traders affect the price in period 0? What happens if $\rho = 0$?

6. This problem asks you to show that with some natural variants on the approach to modeling performance-based risk in Problem 4, consumption is not linear in the shocks, which renders the model intractable.

a. Consider the model in Problem 4. Suppose, however, that the representative hedge fund manager, rather than receiving a payment or incurring a cost in period 1, is forced to sell quantity $b(E_0[P_1] - P_1)H, b > 0$, of the risky asset in period 1. Show that in this case, the manager's consumption is not linear in F_1 .

b. Consider the model in Problem 6 on Problem Set 10. Suppose, however, that A_1 is not exogenous but depends on the success of the period 0 sophisticated investors: $A_1 = \bar{A} + b(P_1 - E_0[P_1])X_0^a, b > 0$. Show that in this case, the consumption of the sophisticated investors born in period 0 is not linear in F_1 .