UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2016
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Problem Set 10
Due in lecture Thursday, November 17

1. (*A simpler approach to agency costs: limited pledgeability*.**)** Consider the model of Section 9.9 (Section 10.2 of the new chapter), with a different friction: there is no cost of verifying output, but the entrepreneur can hide fraction $1 - f$ of the project's output from the investor (with $0 \leq f \leq 1$). Thus the entrepreneur can only credibly promise to repay fraction f of the project's output.

a. Consider a project with expected payoff γ that exceeds $1 + r$. What is the condition for the project to be undertaken?

b. Suppose the condition you found in part (*a*) is satisfied with strict inequality. Is the contract between the investor and the entrepreneur uniquely determined? If so, what is the contract? If not, explain why.

c. Limited pledgeability leads to inefficiency (relative to the case of no frictions) if $\gamma > 1 + r$ but the project is not undertaken. Describe whether each of the following can cause a project with $\gamma > 1 + r$ not to be undertaken:

i. A fall in the entrepreneur's wealth, W.

ii. An increase in the frictions, f.

iii. An increase in idiosyncratic risk. Concretely, suppose that the output of the project is distributed uniformly on $[\gamma - b, \gamma + b]$ rather than uniformly on $[0, 2\gamma]$, and there is an increase in b.

2. a. Show that in the model analyzed in equations (10.a)–(10.i) of Section 10.4 of the new chapter, the unconditional distributions of C_{2t}^{a} and C_{2t}^{n} are not normal.

b. Explain in a sentence or two why the analysis in the text, which uses the properties of lognormal distributions, is nonetheless correct.

3. This problem asks you to tackle a modeling issue whose answer I am unsure of.

At a general level, the idea is: Consider modeling the noise traders in the model analyzed in equations (10.a)–(10.j) of Section 10.4 of the new chapter in terms of shocks to the quantity of the risky asset they demand rather than to their expectations of the price of the risky asset. What happens when we do this? (The rest of the problem provides a specific thought about how one might do this. However, if you want to pursue some other approach to modeling this general idea, that is fine.)

A way of modeling this idea that seems appealing is to model the demand of a representative noise trader as $X_t^a + \omega_t$, where X_t^a is the demand of a representative arbitrageur (see equation [10.c]), and ω_t is an i.i.d., mean-zero, normally distributed shock with variance V_ω. Analyze this model and report what you find, and discuss what you learn from your analysis.

(Intuitively, one might expect: (1) There is some value of V_ω that yields the same equilibrium as that of the model of Section 10.4 (equation [10.d] seems to suggest this – but note that V is not a primitive parameter of the model); (2) That V_ω is an increasing function of V_η (intuitively, both a higher V_ω and a higher V_η correspond to "noisier" noise traders); and (3) For any V_ω, there is also an equilibrium where $P_t = 1$ for all t (see footnote 4 in the new chapter). However, I have worked on this some, and it is not obvious to me that (1), (2), and (3) are in fact all true.)

4. A reduction in an entrepreneur's wealth is likely to increase the agency costs associated with obtaining financing for the entrepreneur's project because:

A. The entrepreneur is in a weaker bargaining position with respect to outside investors.

B. The entrepreneur will spend less on obtaining outside certification of the soundness of his or her project.

C. The entrepreneur's incentives to devote effort on dimensions that outside investors cannot monitor will be lower.

D. Because the entrepreneur needs more outside funding, he or she is likely to need funds from wealthier investors, who will face higher marginal tax rates and so require higher returns.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ANSWERS WILL GENERALLY NOT BE PROVIDED)

5. Consider the model of investment under asymmetric information in Section 10.2 of the new chapter. Suppose that initially the entrepreneur is undertaking the project, and that $(1 + r)(1 - W)$ is strictly less than R^{MAX}. Describe how each of the following affect D:

a. A small increase in W.

b. A small increase in r.

c. A small increase in c.

d. Instead of being distributed uniformly on $[0,2\gamma]$, the output of the project is distributed uniformly on $[\gamma - b,\gamma + b]$, and there is a small increase in b.

e. Instead of being distributed uniformly on $[0,2\gamma]$, the output of the project is distributed uniformly on $[b,2\gamma + b]$, and there is a small increase in b.

6. Consider the following variant on the model analyzed in equations (10.a)–(10.j) of Section 10.4 of the new chapter. There are three periods, denoted 0, 1, and 2. There are two assets. The first is a safe asset in perfectly elastic supply. Its rate of return is normalized to zero: one unit of the economy's single good invested in this asset in period 0 yields one unit of the good for sure in period 1, and one unit of the good invested in this asset in period 1 yields one unit for sure in period 2. The second is a risky asset. Its payoff, which is realized in period 2, is $1 + F_1 + F_2$, where F_t is distributed normally with mean 0 and variance V_t^F. F_1 is observed in period 1, and F_2 is observed in period 2. This asset is in zero net supply. Thus equilibrium requires that the sum across agents of the quantity of the asset demanded is zero.

There are two types of traders. The first are noise traders. They demand quantity N_0 of the risky asset in period 0, and $N_0 + N_1$ in period 1, where N_1 is distributed normally with mean 0 and variance V_1^N (N_0 is exogenous). F_1, F_2, and N_1 are independent. The second are arbitrageurs. A_0 are born in period 0, and A_1 are born in period 1. They live for two periods (0 and 1 for those born in period 0; 1 and 2 for those born in period 1). They care only about consumption in the second period of their life, and have utility $U(C) = -e^{-2\gamma C}, \gamma > 0$. They have no initial wealth.

a. Consider first period 1.

i. Consider a representative arbitrageur born in period 1. What is his or her second-period consumption as a function of P_1, F_1, and F_2, and his or her purchases of the risky asset, X_1^a? What is the mean and variance of his or her second-period consumption as a function of P_1, F_1, X_1^a, and V_t^F? What is the first-order condition for his or her choice of X_1^a?

ii. What is the condition for equilibrium in period 1?

iii. Use the results in (i) and (ii) to find an expression for $P_1 - (1 + F_1)$, the departure of the price in period 1 from its fundamental value.

iv. Do your results support the statement in the text that greater fundamental risk mutes sophisticated investors' willingness to trade to offset departures of asset prices from fundamentals, and so leads to larger departures of asset prices from fundamentals?

b. Now consider period 0.

i. What is the first-order condition for X_0^a (purchases of the risky asset by the representative sophisticated investor) in terms of $E_0[P_1]$ and $\text{Var}(P_1)$ and the parameters of the model?

ii. Use the results from part (*a*) to find $E_0[P_1]$ and $\text{Var}(P_1)$ in terms of exogenous parameters.

iii. Use the results in (i) and (ii) to find an expression for $P_0 - 1$, the departure of the price in period 0 from its fundamental value.

iv. Do increases in fundamental risk (V_1^F and V_2^F) increase departures of asset prices from fundamentals? Do increases in noise-trader risk? Are there interactions—that is, does an increase in noise-trader risk increase, decrease, or have no effect on the effect of fundamental risk?