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Preliminary – please do not circulate!

10.4 Excess Volatility

In the models of financial markets we have considered so far – the partial-equilibrium models of consumers' asset allocation and the determinants of investment in Sections 8.5 and 9.7, and the general-equilibrium model of Section 10.1 – asset prices equal their fundamental values. That is, they are the rational expectations, given agents' information and their stochastic discount factors, of the present value of the assets' payoffs. This case is the natural benchmark. It is generally a good idea to start by assuming rationality and no market imperfections. Moreover, many participants in financial markets are highly sophisticated and have vast resources at their disposal, so one might expect that any departures of asset prices from fundamentals would be small and quickly corrected.

On the other hand, there are many movements in asset prices that, at least at first glance, cannot be easily explained by changes in fundamentals. And perfectly rational, risk neutral agents with unlimited resources standing ready to immediately correct any unwarranted movements in asset prices do not exist. Thus there is not an open-and-shut theoretical case that asset prices can never differ from fundamentals.

This section is therefore devoted to examining the possibility of such departures. We start by considering a model, due to DeLong, Shleifer, Summers, and Waldmann (1990), that demonstrates what is perhaps the most economically interesting force making such departures possible. DeLong et al. show departures of prices from fundamentals can be self-reinforcing: the very fact that there can be departures creates a source of risk, and so limits the willingness of rational investors to trade to correct the mispricings. Indeed, they develop a model in which the presence of some traders who act on the basis of incorrect beliefs has the effect that the price of an asset is not equal to its fundamental value even though its payoffs are certain and some investors are fully rational. After examining DeLong et al.'s model in detail, we will turn to other factors that limit the forces correcting departures of asset prices from fundamentals and discuss the macroeconomic implications of the possibility of excess volatility. Section 10.5 considers some empirical evidence.

Assumptions

DeLong et al. consider an economy with two seemingly identical assets. One unit of either asset pays a constant, known amount $r > 0$ of the economy's single good each period. Where the two assets differ is in their supply. The first asset, which we refer to as safe, can be converted into one unit of the economy's good at any time, and one unit of the good can be converted into the asset. This ensures that its price (in units of the good) is always 1. If not, agents could earn immediate riskless profits by selling the asset for goods and converting the proceeds into the asset (if its price exceeded 1), or buying the asset for goods and converting it into goods (if its price was less than 1). In contrast, the other asset, which we refer to as risky, cannot be created or destroyed. Thus its supply is equal to a constant, which for simplicity we normalize to 1.

The economy is an overlapping-generations economy with two types of agents. They are the same in several ways. They live for two periods, are price-takers, and value consumption only in the second period of life. For tractability, each agent's utility is of the constant-absolute-risk-aversion form, $U(C) = -e^{-2\gamma C}, \gamma > 0$, where C is the agent's consumption in his or her second period. Finally, each agent has the same amount of first-period income, which is constant over time. Since there is nothing special about whether consumption is positive or negative when agents have constant-absolute-risk-aversion utility, we normalize that income to zero.

Where the types differ is in their beliefs about the returns to the risky asset. The first type have rational expectations. That is, they correctly perceive the distribution of returns from holding the asset. We refer these agents as arbitrageurs. The second type misestimate the mean return on the asset. In particular, in period t, the entire distribution of the price of the asset in period $t + 1$, p_{t+1}, perceived by each agent of this type is shifted relative to the true distribution by an amount η_t. η is independent over time and normally distributed with mean zero and variance V_η. We refer to these agents as *noise traders*, which is the conventional term for agents who trade in financial markets based on information unrelated to fundamentals. The fraction of noise traders in each generation is f, where $0 \leq f \leq 1$. For simplicity, population growth is assumed to be zero, and the size of each generation is set to 1.

Analyzing the Model

The assumptions of the model are chosen so that there is a stationary equilibrium where the price of the asset is linear in the shocks and where the distribution of agents' second-period consumption, conditional on their first-period information, is normal. Two assumptions are central to this result. The most obvious is that the only shock in the model (the shift in the noise traders' beliefs) is normally distributed with the same distribution each period. The other is that utility takes the constant-absolute-risk-aversion utility form. To see why this assumption leads to tractable results, note that if C is normally distributed with mean μ and variance W, then $-2\gamma C$ is normal with mean $-2\gamma\mu$ and variance $4\gamma^2 W$. Thus (by the properties of lognormal distributions), expected utility, $\mathrm{E}[-e^{-2\gamma C}]$, is $2\gamma\mu - 2\gamma^2 W$, which is proportional to $\mu - \gamma W$. That is, the combination of normally distributed consumption and constant-absolute-risk-aversion utility implies that agents act as if they have linear preferences over the mean and variance of consumption. This causes outcomes to be linear in the shock.

Our strategy will therefore be to look for a stationary equilibrium where the price of the asset is linear in the shock. To find such an equilibrium, consider first the arbitrageurs. Let $\mathrm{E}_t[p_{t+1}]$ denote the rational expectation of p_{t+1} (the price of the risky asset in period $t + 1$) given the information available at t, and let V denote the variance of $p_{t+1} - \mathrm{E}_t[p_{t+1}]$. If an arbitrageur in period t buys X_t^a of the risky asset, he or she must hold $-p_t X_t^a$ of the safe asset (recall that first-period income is assumed to be zero). Both assets pay r per unit in period $t + 1$. The risky asset is sold at a price of p_{t+1}, while the safe asset is sold at a price of 1. Thus the agent's second-period consumption is

$$
\begin{aligned}
C_{2t}^a &= r(X_t^a - p_t X_t^a) + p_{t+1} X_t^a - p_t X_t^a \\
&= [r + p_{t+1} - (1+r)p_t] X_t^a.
\end{aligned}
\tag{10.a}
$$

Equation (10.a) implies that given the information available at time t, C_{2t}^a has mean $[r + \mathrm{E}_t[p_{t+1}] - (1+r)p_t] X_t^a$ and variance $(X_t^a)^2 V$. Note also that the only variable in the expression for C_{2t}^a that is uncertain as of period t is p_{t+1}. Thus if p_{t+1} is linear in η_{t+1}, the distribution of C_{2t}^a (given the information available at t) is normal.

We saw above that the agent's expected utility is proportional to the mean of consumption minus γ times the variance. Thus the first-order condition for the level of X_t^a that maximizes the agent's expected utility is

$$[r + \mathrm{E}_t[p_{t+1}] - (1+r)p_t] - 2\gamma X_t^a V = 0. \tag{10.b}$$

Solving for X_t^a yields

$$X_t^a = \frac{r + \mathrm{E}_t[p_{t+1}] - (1+r)p_t}{2\gamma V}. \tag{10.c}$$

Since all period-*t* arbitrageurs are the same, each purchases this quantity of the risky asset.

The analysis of the representative noise trader's behavior is identical, except that $\mathrm{E}_t[p_{t+1}]$ is replaced by the agent's incorrect belief about the mean of p_{t+1}, which is $\mathrm{E}_t[p_{t+1}] + \eta_t$. His or her demand is therefore

$$X_t^n = \frac{r + \mathrm{E}_t[p_{t+1}] + \eta_t - (1+r)p_t}{2\gamma V}. \tag{10.d}$$

When the economy enters period *t*, the fixed supply of the risky asset is held by old agents, who sell their holdings regardless of the price. Thus equilibrium requires that the sum of the demands of the 1 – *f* arbitrageurs and the *f* noise traders equals the fixed supply, which we have set to 1:

$$(1-f)X_a^t + fX_n^t = 1. \tag{10.e}$$

Substituting in expressions (10.c) and (10.d) and solving for p_t gives

$$p_t = \frac{r + \mathrm{E}_t[p_{t+1}] + f\eta_t - 2\gamma V}{1+r}. \tag{10.f}$$

Applying (10.f) to future periods (and taking expectations of both sides as of period *t*) implies that $\mathrm{E}_t[p_{t+1}] = (r + \mathrm{E}_t[p_{t+2}] - 2\gamma V)/(1+r)$, $\mathrm{E}_t[p_{t+2}] = (r + \mathrm{E}_t[p_{t+3}] - 2\gamma V)/(1+r)$, and so on.[3] Repeated substitution into (10.f) therefore gives us:

$$p_t = \left(\frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \cdots\right)(r - 2\gamma V) + \lim_{n\to\infty} \frac{\mathrm{E}_t[p_{t+n}]}{(1+r)^n} + \frac{f\eta_t}{1+r}. \tag{10.g}$$

The fact that we are focusing on stationary equilibria implies that the mean of *p* is constant over time, and thus that $\lim_{n\to\infty} \frac{\mathrm{E}_t[p_{t+n}]}{(1+r)^n}$ is zero. In addition, the infinite sum in (10.g) simplifies to $1/r$. Thus, (10.g) implies:

$$p_t = 1 - \frac{2\gamma V}{r} + \frac{f\eta_t}{1+r}. \tag{10.i}$$

The last step in solving the model is to rewrite *V*, the variance of $p_{t+1} - \mathrm{E}_t[p_{t+1}]$, in terms of primitive parameters. The only stochastic term in (10.i) is $f\eta_t/(1+r)$. Thus, (10.i) implies $V = [f^2/(1+r)^2]V_\eta$. Substituting this expression into (10.i) gives us our final equation

[3] Recall that we defined *V* as the variance of $p_{t+1} - \mathrm{E}_t[p_{t+1}]$. Thus, this step implicitly assumes that the variances of $p_{t+2} - \mathrm{E}_{t+1}[p_{t+2}]$, $p_{t+3} - \mathrm{E}_{t+2}[p_{t+3}]$, and so on are all equal to *V*. Equation (10.i) shows that this assumption is correct in the equilibrium that we find. Another approach would be to appeal to the assumption of stationarity and the fact that no past variables appear in (10.f) to conjecture that $\mathrm{E}_{t-1}[p_t] = \mathrm{E}_{t-1}[p_{t+1}] = \mathrm{E}_t[p_{t+1}]$. Taking expectations of both sides of (10.f) as of $t-1$ and using the conjecture then gives $\mathrm{E}_t[p_{t+1}] = 1 - (2\gamma V/r)$. Substituting this expression into (10.f) then yields (10.i), and (10.i) shows that the conjecture is correct.

for the equilibrium price in a given period:

$$p_t = 1 - \frac{2\gamma}{r}\frac{f^2}{(1+r)^2}V_\eta + \frac{f\eta_t}{1+r}. \qquad (10.j)$$

Note that the price is linear in η. It follows that the distributions of the consumption of agents born at t, conditional on the information available at t, are normal (see [10.a]). Equation (10.j) also implies that the distribution of p is the same each period. Thus we have found an equilibrium of the form we were looking for.

Discussion

The model's key implication is that the price of the asset is risky despite the fact that there is no uncertainty about its payoffs – everyone knows that it will pay r each period with certainty. The reason is that the fluctuations in the beliefs of the noise traders are themselves a source of risk. Rational traders considering buying the asset must be concerned not only about its dividend payments, but also about the price at which they will sell the asset. Even if the sentiments of the noise traders are pushing down the asset's current price, a rational trader who buys the asset at that low price faces the possibility that sentiments could deteriorate further by the time he or she needs to sell the asset, and so depress the price even more. Thus, "noise-trader risk" limits the willingness of rational but risk averse investors to trade to offset departures of prices from fundamentals. As Delong et al. put it, noise traders "create their own space."[4]

[4] One might expect that even if there is an equilibrium where asset prices fluctuate in response to noise traders' sentiments, there would be another where they do not. After all, if rational traders know that the price of the "risky" asset always equals 1, the asset is riskless, and so their demand for the asset is perfectly elastic. What rules out this potential equilibrium in DeLong et al.'s model is their assumption that what is exogenous and constant is the variance of the error in noise traders' beliefs about the mean of next period's price, rather than the variance of the shock to the quantity of the risky asset they demand. As a result, in the proposed equilibrium where the asset's price

The short horizons of the rational traders are critical to this finding. If they had infinite horizons, they could hold the asset indefinitely, and so the selling price would be irrelevant. As a result, departures of the asset's price from its fundamental value would not be possible. The assumption of limited horizons is reasonable, however. The ultimate holders of assets are individuals. They are likely to need to sell their assets at some point, for example to pay for consumption in retirement or to smooth their consumption in the face of fluctuations in labor income. In addition, as we discuss below, there are forces that may make the horizons of portfolio managers shorter than those of the underlying asset-holders.

The model implies not only that the price of the risky asset fluctuates, but also that on average it is less than its fundamental value of 1 (see [10.j]). This result is a direct consequence of the noise-trader risk: the asset is riskier than is warranted by fundamentals, and so agents, who are risk averse, are on average willing to pay less for it.

Finally, the model has implications about the effects of changes in the parameters. Most are unsurprising. Equation (10.j) shows that the mean departure of prices from fundamentals is larger when agents are more risk averse, when there are more noise traders, and when the variance of sentiment shocks is greater. And it is smaller when r is larger, which corresponds to a larger fraction of the present value of the payoff to the asset being paid in the next period. Similarly, the effect of a given shift in sentiment is larger when there are more noise traders and smaller when r is greater.

The only implications about the effects of the parameters that may be surprising concern two parameters that do not appear in the last term of (10.j): neither risk aversion (γ) nor the

is constant and equal to 1, noise traders' demand each period would be infinite (either positive or negative). This discussion implies that the result that there is not a second equilibrium where the noise traders do not cause prices to depart from fundamentals is not general. Rather, DeLong et al. make a particular assumption that eliminates this equilibrium, which allows them to focus on the interesting case where there are departures from fundamentals.

variance of sentiment shocks (V_η) influences how a given shift in sentiment affects the price of the asset. The reason is that the sentiment shocks correspond to a given change in expectations of next period's price rather than in the quantity demanded. As a result, higher risk aversion and a greater variance of sentiment shocks mute not only the arbitrageurs' willingness to trade to correct mispricings, but also noise traders' willingness to trade on the basis of their sentiments (see equation [10.f]).

Other Factors Limiting Arbitrageurs' Willingness to Trade to Correct Mispricings

Researchers have identified two factors in addition to noise-trader risk that mute the extent to which sophisticated investors are willing to trade to move asset prices back toward fundamentals if they depart from them. Like noise-trader risk, these forces make it easier for asset prices to differ from fundamentals.

The most obvious additional factor is "fundamental risk." Suppose an asset is undervalued given currently available information. An investor who buys the asset faces the risk that its price will fall because of the arrival of new information about its future payoffs. That is, the price of an undervalued asset can fall not just because its price falls further below fundamentals (as in our model of noise-trader risk), but also because the fundamental value of the asset declines.

The second is "agency risk" (Shleifer and Vishny, 1997). Many of the investors who are most likely to trade to exploit mispricings rely mainly on funds obtained from others. If the funders of these investors base their assessment of the investors' abilities partly on their short-run performance, they may withdraw their funds – and so force investors to sell undervalued assets – precisely in situations where the mispricings have worsened in the short run, and so the prices of the assets have fallen. That is, the fact that sophisticated traders are often acting as agents of less knowledgeable individuals can force the sophisticated investors to behave as if they have short horizons. Notice that in contrast to the baseline model of noise-trader risk, in this case the sophisticated investors' short horizons arise endogenously, and only occur when the departure of an asset's price from fundamentals becomes larger.[5]

Problems 10.a and 10.b develop a model where noise-trader risk, fundamental risk, and agency risk are present together.[6]

Macroeconomic Implications

Our analysis of excess volatility shows that the forces working against possible mispricings are not infinitely strong. Risk neutral agents with unbounded funds at their disposal do not exist. As a result, rational investors' willingness to trade to correct a possible mispricing is inherently limited. Our analysis identifies three specific factors that weaken the forces working to mute mispricings: noise-trader risk, fundamental risk, and agency risk.

But the fact that departures of prices from fundamentals are possible does not prove that they are important for the economy. A first question is whether they can be quantitatively large. A simple calculation suggests that they can if two key conditions are met: they involve assets whose prices are relatively volatile, and they are on average fairly persistent. Consider a class of assets that rational investors estimate is overvalued relative to fundamentals by 30 percent. Suppose first that the asset class is all equities and that on average a tenth of a mispricing

[5] The essence of agency risk is nicely summarized by a statement that is sometimes attributed (erroneously) to Keynes: "The market can remain irrational longer than you can remain solvent."
[6] A fourth factor potentially limiting sophisticated investors' willingness to trade to correct mispricings is "model-based risk": arbitrageurs cannot be certain that their estimates of fundamental values are in fact the best estimates given the available information. This risk can be thought of as just a subtle form of fundamental risk: the fundamental value of the asset may turn out to be less than currently expected not just because of the arrival of conventional types of news, but also because of new information showing that the model that sophisticated investors were using to estimate fundamentals was incorrect.

disappears over a year. Recall from Section 8.5 that the average annual excess return of equities over a safe interest rate is about 6 percentage points, and that the standard deviation of the excess return is about 17 percentage points. Then rational investors believe that the expected excess return over the next year is 3 percentage points, and (if returns are normal) that there is a 43 percent chance rather than the usual 50 percent chance that the excess return will be greater than its long-term average. The optimal response to this information is simply to hold fewer equities than usual, not to make large trades against them. Thus the forces acting to prevent such a mispricing would not be particularly strong. On the other hand, if the mispricing concerns a class of assets whose excess return has a standard deviation of 5 percentage points and on average one-third of the mispricing is corrected in a year, then rational investor believe that the expected excess return over the coming year is −4 percent, and that the chance of the excess return being negative is about 80 percent. Thus, the incentives to trade against the mispricing are vastly larger, and so the forces preventing such a mispricing are vastly stronger.[7]

A second question about whether departures of asset prices from fundamentals can be important to the economy is whether substantial mispricings can have substantial macroeconomic effects. Again the answer appears to be yes: large movements in asset prices appear to have large effects on the composition and level of economic activity. Theoretically, we know that asset prices affect both investment and consumption. In the q theory model of investment, the market prices of various capital goods are critical inputs into firms' investment decisions. And as described in Section 10.2, increases in asset prices reduce distortions in financial markets, and so raise investment. Likewise, the permanent-income hypothesis implies

[7] Endogenous information acquisition is likely to make these effects somewhat self-reinforcing. Information about mispricings of assets with low volatility where departures of prices from fundamentals are corrected quickly is much more valuable than information about mispricings of assets with high volatility where departures from fundamentals are highly persistent. Individuals interested in profiting from trading against mispricings therefore concentrate on acquiring the first type of information. As a result, the pool of agents who know enough to identify large, fairly persistent mispricings and who trade against them is endogenously reduced.

that increases in wealth resulting from higher asset prices raise consumption. And if households are liquidity constrained, increases in wealth relax the constraints, and so potentially raise consumption by more than implied by the permanent-income hypothesis. In the extreme, a household that is at a corner solution for its choice of consumption may raise its consumption one-for-one with increases in wealth (see equation [8.46] in Section 8.6).

Empirically, the huge run-up in the prices of technology stocks in the late 1990s appears to have led to a large increase in investment in fiber optic cable, capital goods of various internet start-ups, and so on, and to have been an important driver of the boom in overall economic activity in that period. Likewise, the enormous increase in house prices in the early 2000s appears to have caused very high housing investment. More interestingly, there is evidence that it also led to large increases in consumption, particularly among low-income homeowners, as homeowners tapped their new wealth (Mian and Sufi, 2014). As with the 1990s dot-com boom, these forces again had a notable impact on the overall economy.

Thus, there are strong reasons to believe that substantial departures of asset prices from fundamental are possible, and that if they occur, they are likely to have significant macroeconomic consequences. But that does not tell us whether such mispricings occur. The next section therefore turns to empirical evidence on that issue.