UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2016
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Problem Set 9
Due in lecture Thursday, November 10

1. In the firm optimization problem in the q-theory model, the transversality condition rules out:

A. Paths where the firm is violating its budget constraint by going further and further into debt.

B. Paths where investment does not satisfy 1 + C'(I(t)) = q(t).

C. Paths where the firm is constantly increasing its investment even though the profitability of capital is constantly falling.

D. Paths where investment approaches zero.

2. In the q-theory model where the initial value of K exceeds its long-run equilibrium value, as the economy moves toward the long-run equilibrium:

A. The $\dot{q} = 0$ locus is shifting to the right and the $\dot{K} = 0$ locus is shifting down.

B. The $\dot{q} = 0$ locus is shifting to the right and the $\dot{K} = 0$ locus is not shifting.

C. The $\dot{K} = 0$ locus is shifting down and the $\dot{q} = 0$ locus is not shifting.

D. None of the above.

3. Romer, Problem 9.7.

4. Romer, Problem 9.12, parts (b), (c), and (d).

5. Consider the model of the first section of the notes posted for lecture on 11/8. Assume that utility is logarithmic, that $\beta = 1$, and that there are only two states, each of which occurs with probability one-half. Finally, there is only one investment project. It pays R_G in state G and R_B in state B, with $R_G > R_B > 0$. We will refer to G as the "good" state and B as the "bad" state.

a. What are the equilibrium conditions?

b. What are C_1, K, C_2^G, C_2^B, q_G, and q_B?

c. Now suppose a new investment project is discovered. It pays off only in one state of the world. Let $R_{NEW} > 0$ denote its payoff in that state.

i. What is the condition for there to be strictly positive investment in the new project?

ii. Assume the condition in (i) is satisfied. What are the equilibrium levels of K (investment in the old project) and K_{NEW} (investment in the new project)?

iii. Suppose the state in which the new project pays off is B. What is the condition for C_2^B to be greater than or equal to C_2^G, so that it is no longer reasonable to describe B as the "bad" state?

EXTRA PROBLEMS (NOT TO BE HANDED IN/ANSWERS WILL GENERALLY NOT BE PROVIDED)

6. ***The saddle-path of the q-theory model.*** Consider the two equations of the q-theory model,

$$\dot{q}(t) = rq(t) - \pi\big(K(t)\big), \qquad \dot{K}(t) = f\big(q(t)\big).$$

a. Define the steady state of the model, $(\bar{q}, \bar{K})$. Show that the model's linear (Taylor) approximation in the neighborhood of the steady state takes the form:

$$[\dot{q}, \dot{K}]' \approx G[q - \bar{q}, K - \bar{K}]',$$

where G =

A	B
C	0

Be sure to show how A, B, and C depend on exogenous parameters, the steady-state values of q and K, and/or the properties of $\pi(\cdot)$ and $f(\cdot)$.

b. Show that the characteristic roots of the preceding 2x2 matrix are:

$$\lambda_1, \lambda_2 = \frac{r \mp \sqrt{r^2 - 4f'(1)\pi'(\bar{K})}}{2},$$

where $\lambda_1 > 0$ and $\lambda_2 < 0$. Please indicate why the second condition holds.

c. Show that the eigenvectors of the matrix G are proportional to the matrix

X =

$\lambda_1/f'(1)$	$\lambda_2/f'(1)$
1	1

d. Define $[\tilde{q}, \tilde{K}]' \equiv X^{-1}[q - \bar{q}, K - \bar{K}]'$, and note that this implies that $[\dot{\tilde{q}}, \dot{\tilde{K}}]' = X^{-1}[\dot{q}, \dot{K}]'$. Explain how that change of variables enables us to write the solution of our differential equation system in the form $[\tilde{q}(t), \tilde{K}(t)]' = [\tilde{q}(0)e^{\lambda_1 t}, \tilde{K}(0)e^{\lambda_2 t}]'$ for arbitrary initial conditions $\tilde{q}(0)$ and $\tilde{K}(0)$.

e. From this last relationship deduce that:

$$q(t) - \bar{q} = \tilde{q}(0)(\lambda_1/f'(1))e^{\lambda_1 t} + \tilde{K}(0)(\lambda_2/f'(1))e^{\lambda_2 t},$$

$$K(t) - \bar{K} = \tilde{q}(0)e^{\lambda_1 t} + \tilde{K}(0)e^{\lambda_2 t}.$$

f. Recalling that $\lambda_1 > 0$ and $\lambda_2 < 0$, identify the initial condition that will ensure the economy is on the convergent saddle-path in the usual phase diagram with K on the horizontal axis and q on the vertical axis.

g. For our linear approximation above, express the (linear) equation for the saddle-path in the form

$$q(t) - \bar{q} = \Omega[K(t) - \bar{K}]$$

for an appropriate constant slope $\Omega < 0$. Be sure to show how Ω depends on the model parameters, the steady-state values of q and K, and/or the properties of $\pi(\cdot)$ and $f(\cdot)$.

h. Recall that

$$\lambda_2 = \frac{r - \sqrt{r^2 - 4f'(1)\pi(\bar{K})}}{2}.$$

Discuss which parameters and steady state values affect the slope of the saddle-path. How do they impact the slope? Why?

7-8. Romer, Problems 9.13, 9.14.