UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2016
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Problem Set 8
Due in lecture Thursday, November 3

1. Consider Problem 1 on Problem Set 7. For part (g), please submit your code.

f. Consider the following seemingly small variation on part (b) of that problem. Choose an N, and define e = 200/N. Now, assume that Y can take on only the values 0, e, 2e, 3e, …, 200, each with probability 1/(N + 1). Likewise, assume that c can only take on the values 0, e, 2e, 3e, …., and find the $V^n(x)$'s only for x equal to 0, e, 2e, … up to some upper bound B that you choose (and assume that $V^n(x) = V^n(B)$ for x > B).

Show (analytically, not numerically) that value function iteration using this numerical algorithm will converge to $V(x) = -\infty$ for all x.

(Hint: If you get stuck, try it with N = 2, B = 300.)

(The message here is that numerical analysis isn't as easy as it seems. Both the approach in part (b) and the approach here probably seem reasonable at first glance. But they give completely different answers.)

g. Use your analysis from last week to see how a one-time income shock affects the paths of consumption and wealth starting from different situations. Specifically, plot, as a function of time, the difference between the paths of C for a household with a realized path of income of {10,100,100,100,…} and a household with a realized path of income of {100,100,100,100,…}:

i. In the case where both households enter the initial period with A = 10.
ii. In the case where both households enter the initial period with A = 200.
iii. Discuss your results.

(Notes: (1) Recall that $X_t = A_t + Y_t$. So, for example, if the household enters the initial period with A = 10 and its income that period is 10, its initial X is 20. (2) As in the rest of the problem, the household believes that its income is distributed uniformly on [0,200] each period. Thus, the V(•) and c(•) you found last week still apply.)

2. Consider an economy that lasts for two periods and that consists of equal numbers of two types of agents, Type A and Type B. The objective function of a representative agent of Type *i* is

$$C_1^i + \beta E\left[C_2^i - \frac{1}{2}a(C_2^i)^2\right], \quad a > 0.$$

where C_t^i is the consumption of an agent of Type *i* in period t. Assume that the C_2^i's are always in the range where marginal utility is positive.

Agents of Type *i* receive an endowment of W_1^i in period 1 and W_2^i in period 2. The W_1^i's are certain and the W_2^i's are uncertain.

Endowments cannot be stored or saved in any way. Thus equilibrium requires $C_1^A + C_1^B = W_1^A + W_1^B$ and $C_2^A + C_2^B = W_2^A + W_2^B$.

a. Suppose the only asset that can be traded is a riskless bond. Specifically, consider an asset that will pay 1 unit for sure in period 2.

i. Set up the problem of an agent of Type *i* choosing how much of the asset to buy. The agent takes P, the price of the asset in period 1 in units of period-1 endowment, as given. The amount bought can be positive or negative (that is, the agent can buy or sell the asset).

ii. Find the demand of an agent of Type *i* for the asset as a function of P and of any relevant parameters (for example, a, β, W_1^i, and the mean and variance of W_2^i).

iii. What is the equilibrium price of the asset? (Hint: What must the sum of the quantities of the asset demanded by the two types of agents be for the market to be in equilibrium?)

b. Suppose agents cannot trade a safe asset, but can trade two risky assets, A and B. The payoff to Asset *i* is W_2^i. Let P_i denote the period-1 price of Asset *i* in units of period-1 endowment. (Thus, if an agent of Type *i* buys Q_A^i of Asset A and Q_B^i of Asset B, his or her consumption is $W_1^i - P_A Q_A^i - P_B Q_B^i$ in period 1, and $W_2^i + Q_A^i W_2^A + Q_B^i W_2^B$ in period 2.)

i. Set up the problem of an agent of Type *i* choosing how much of each of the two assets to buy. The agent takes the prices of the assets in period 1 as given. (As in part (a), the amounts bought can be positive or negative.)

ii. Find the first-order conditions for the problem you set up in part (b)(i).

iii. Assume $W_1^A = W_1^B$, and that W_2^A and W_2^B have the same distribution as one another and are independent. If $P_A = P_B$, will a Type-A agent demand more of Asset A or of Asset B? (A good logical explanation is enough.)

iv. Continue to make the assumptions in part (b)(iii). Get as far as you can in describing the equilibrium quantities ($Q_A^A, Q_B^A, Q_A^B,$ and Q_B^B). (As in part (iii), a good logical argument is enough.)

3. Consider the q-theory model. Assume that initially the economy is in steady state. Let K^{*OLD} denote the steady-state value of K, and let $\pi^{OLD}(\bullet)$ denote the $\pi(\bullet)$ function.

At some time, which we will call time 0 for simplicity, there is a permanent, unexpected shift of the $\pi(\bullet)$ function. The new function is $\pi^{NEW}(K) = A$ for all K, where $A > \pi^{OLD}(K^{*OLD})$.

a. How, if at all, do the $\dot{q} = 0$ and $\dot{K} = 0$ loci change at t = 0?

b. How, if at all, do q and K change at t = 0?

c. Describe the behavior of q and K after t = 0.

Explain your answers.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ANSWERS WILL GENERALLY NOT BE PROVIDED)

4. Consider Problem 1 on Problem Set 7:

h. Tweak something about the model (the obvious candidates are the utility function, β, 1 + r, and the distribution of Y) and find the new V(•) and c(•). Discuss how the change in assumptions changes the results, and explain the intuition.

i. Part (b) had you use a very primitive way of tackling the problem numerically. How might one do better? (Some candidates might involve interpolation or extrapolation, or not making the points you consider equally spaced.)

5. Romer, Problem 9.6.