UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2016
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Problem Set 6
Due in lecture, Thursday, October 20

1. Recall the problem from the midterm on progressive capital taxation and household optimization. Here are two additional parts of that problem.

f. In the absence of capital taxation, the household's budget constraint would be

$$\int_{t=0}^{T} e^{-R(t)}C(t)dt \le A(0) + \int_{t=0}^{T} e^{-R(t)}w(t)dt, \quad \text{where } R(t) \equiv \int_{s=0}^{t} r(s)ds.$$

By writing the budget constraint this way, it is possible to solve the household's maximization problem using a Lagrangian rather than the Hamiltonian. Explain in a few sentences whether or not it is possible to solve the household's optimization problem using that approach when there is progressive capital taxation.

g. The model implies:

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \tau\big(A(t)\big) - A(t)\tau'\big(A(t)\big) - \rho}{-C(t)U''(C(t))/U'(C(t)}.$$

Now suppose we embed this in the Ramsey-Cass-Koopmans model. To make this a little easier, assume that households are infinitely-lived (rather than finitely-lived, as assumed on the midterm), that $n = g = 0$, that tax revenues are rebated lump-sum to households, and that $\tau(\bullet) > 0, \tau'(\bullet) > 0, \tau"(\bullet) > 0$.

How, if at all, will going from not having capital taxation to having progressive capital taxation affect the $\dot{c} = 0$ and/or $\dot{k} = 0$ loci?

2. Romer, Problem 4.10.

3. The basic model of consumption under uncertainty (with quadratic utility, and uncertainty only about labor income) implies:

A. The change in income will not be predictable on the basis of past changes in consumption.
B. The change in consumption will not be predictable on the basis of past changes in income.
C. The change in consumption will not be correlated with the current change in income.
D. (A) and (B).
E. (A) and (C).
F. (B) and (C).

4. (Consumption with state-contingent goods.) Consider a consumer whose labor income (which he or she takes as exogenous) is uncertain. Specifically, the consumer's labor income in state s in period t is Y_{st}. The probability that the state in period t is s is π_{st}. Thus, for each t, $\sum_s \pi_{st} = 1$. The realization of the state each period is independent of the realization in all other periods.

The consumer seeks to maximize $E\left[\sum_t \frac{1}{(1+\delta)^t} U(C_t)\right], U'(\cdot) > 0, U"(\cdot) < 0$. The consumer can purchase state-contingent goods and sell his or her state-contingent income. The price of consumption in period t in state s is p_{st}. Thus, we can write the consumer's objective function as $\sum_t \sum_s \pi_{st} \frac{1}{(1+\delta)^t} U(C_{st})$, and his or her budget constraint as $\sum_t \sum_s p_{st} C_{st} \le \sum_t \sum_s p_{st} Y_{st}$.

a. Set up the consumer's maximization problem, and find the first-order condition for C_{st}.

b. Consider two states in some period t, s' and s''. Under what conditions is consumption the same in the two states? (That is, under what conditions is $C_{s't} = C_{s''t}$?)

c. Consider state s' in period t' and state s'' in period t''. Under what conditions is $C_{s't'} = C_{s''t''}$?

d. Consider 2 consumers who differ only in their Y_{st}'s. Show or provide a counterexample to the following claim: If Consumer 1's consumption in one period is greater than Consumer 2's consumption in that period, Consumer 1's consumption in each period is greater than Consumer 2's consumption in the same period.

e. Suppose that both consumers have constant relative risk aversion utility, with the same coefficient of relative risk aversion. What, if anything, can one say about how the ratio of Consumer 1's consumption to Consumer 2's consumption behaves over time?

f. In practice, we often see consumption reversals (that is, one consumer initially having consumption higher than another, but later having lower consumption). List 2 or 3 ways the assumptions of this problem could fail that could make such reversals possible; explain each possibility in no more than a sentence.

g. Suppose that in some period, the realization of s is the one that has the highest value of $p_{st}Y_{st}$ for that period for the consumer. How, if at all, will that affect the consumer's consumption in later periods?

EXTRA PROBLEMS (NOT TO BE HANDED IN/ANSWERS WILL GENERALLY NOT BE PROVIDED)

5. Romer, Problem 3.9.

6. Romer, Problem 4.3.

7. A consumer facing income uncertainty whose objective function is $E_0[\sum_{t=0}^{\infty} \beta^t U(C_t)]$ and who can borrow and lend at the risk-free interest rate r will satisfy:

A. $U'(C_t) = (1 + r)\beta U'(C_{t+1})$.
B. $U'(C_t) = (1 + r)\beta E_0[U'(C_{t+1})]$.
C. $E_0[U'(C_t)] = (1 + r)\beta E_0[U'(C_{t+1})]$.
D. $U'(C_t) = (1 + r)\beta E_t[U'(C_{t+1})]$.
E. (A) and (C).
F. (B) and (D).
G. (C) and (D).
H. All of the above.

8. Consider a household that will live from 0 to T choosing its path of consumption to maximize its lifetime utility, which is given by:

$$\int_{t=0}^{T} e^{-\rho t} u(C(t))dt,$$

where u(•) takes the constant-relative-risk-aversion form:

$$u(C) = \frac{C^{1-\theta}}{1-\theta}, \quad \theta > 0.$$

The household has no initial wealth; its labor income is constant and equal to $\bar{Y}, \bar{Y} > 0$; and the real interest rate is constant and equal to $\bar{r}, \bar{r} > \rho$. As usual, the present discounted value of the household's consumption cannot exceed the present discounted value of the its lifetime resources.

a. What is the present value Hamiltonian?

b. Find the conditions that characterize the solution to the household's maximization problem.

c. Sketch the paths of the household's asset holdings and of $\ln C$ from 0 to T. (Note: The problem is not asking you to solve explicitly for asset holdings and $\ln C$ as functions of t.)

9. Romer, Problem 2.2.