UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2016
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Problem Set 5
Due at the start of class, Thursday, October 6

1. (From last year's midterm.) Consider the model of endogenous knowledge accumulation presented in the book and in lecture for the case of $\theta < 1$:

$$Y(t) = (1 - a_L)L(t)A(t), \quad 0 < a_L < 1,$$

$$\dot{A}(t) = B[a_L L(t)]^{\gamma} A(t)^{\theta}, \quad B > 0, \qquad \gamma > 0, \qquad \theta < 1.$$

$$\dot{L}(t) = nL(t).$$

Assume $L(0) > 0$, $A(0) > 0$. As in the usual model, a_L is exogenous and constant.

In contrast to the baseline version of the model, assume that the rate of population growth is a decreasing function of the fraction of workers who are engaged in R&D: $n = n(a_L),\ n'(\bullet) < 0,\ n(\bullet) > 0$. (The idea is that, for some reason, scientists on average have fewer children than other workers.)

Suppose the economy is on a balanced growth path, and that there is a permanent increase in a_L. Sketch the resulting path of $\ln A$ and what that path would have been without the increase in a_L. Explain your answer.

INSTRUCTIONS FOR PROBLEMS 2-7:

– Give the best answer to 5 of the following 6 questions. Note:

– If you wish, you may add a BRIEF explanation of your answer to AT MOST ONE question. In that case, your score on that question will be based on your answer and explanation together. This means that an explanation can either raise or lower a grade.

– If you answer all 6 questions, your score on these questions will be based on your average, not on your 5 best scores.

2. In models where the allocation of resources to R&D is determined by market forces, the inputs that embody different ideas are typically modeled as:

A. Supplied in exogenously determined amounts.
B. Public goods.
C. Perfect substitutes for one another.
D. Imperfect substitutes for one another.

3. One of the empirical issues that Jones addresses in "Time-Series Tests of Endogenous Growth Models" is:

A. Whether population growth is stationary or nonstationary.
B. Whether the growth rate of income per capita is higher in countries with larger populations.
C. The horizon over which investment affects growth.
D. The correlation between the number of scientists and engineers and the saving rate.

4. In the P. Romer model of endogenous technological change, the condition for equilibrium in the allocation of workers between R&D and goods production at time *t* is:

A. The wages in the two sectors at time *t* are equal.

B. The present value of the revenues from an idea created at time *t* equals the wage in the goods-producing sector at time *t*.

C. The marginal product of an idea in creating new ideas equals its marginal product in goods production.

D. The price of using an idea equals $\eta/(\eta - 1)$ times the cost of producing the idea, where η is the elasticity of demand for the input using a given idea.

5. The "accounting" approach to decomposing cross-country income differences described in Section 4.2 of Romer, *Advanced Macroeconomics*, <u>fails</u> to assign to human capital:

A. Differences in income stemming from differences in the quality of schooling.

B. Any impact of human capital on income that operates through externalities.

C. The fact that when human capital raises income, if the saving rate does not change then the quantity of saving rises, thereby raising the stock of physical capital.

D. (A) and (B).

E. (A) and (C).

F. (B) and (C).

G. (A), (B), and (C).

H. None of the above.

6. Of the following possible regression results concerning the elasticity of long-run output with respect to the saving rate, the one that would provide the best evidence that differences in saving rates are not important to cross-country income differences is:

A. A point estimate of 5, with a standard error of 2.

B. A point estimate of 0.1, with a standard error of 0.01.

C. A point estimate of 0.001, with a standard error of 5.

D. A point estimate of −2, with a standard error of 5.

7. Consider an economy described by: $\dot{B}(t) = bB(t)$, $\dot{D}(t) = d[cB(t)]^{\omega}D(t)^{\mu}$, $J(t) = [(1-c)B(t)]D(t)$, with $b > 0$, $d > 0$, $0 < c < 1$, $\omega > 0$, $B(0) > 0$, and $D(0) > 0$. This economy will converge to a balanced growth path if and only if:

A. $\mu < 1$.

B. $\mu \leq 1$.

C. $\omega < 1$.

D. $\omega \leq 1$.

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EXTRA PROBLEMS (NOT TO BE HANDED IN / AT MOST A VERY SMALL NUMBER OF ANSWERS WILL BE PROVIDED)

8. Knowledge accumulation may vary in a complicated way over time. This problem asks you to investigate one way that this might occur.

For simplicity, <u>population is constant</u>. Output at time t is given by $Y(t) = (1 - a_L)A(t)L$, where Y is output, a_L is the fraction of the population that is engaged in producing knowledge, A is knowledge, and L is population.

Knowledge accumulation is given by the function: $\dot{A}(t) = B_1 a_L L A(t)^{\theta}$ if $A < A^*$, $\dot{A}(t) = B_2 a_L L$ if $A > A^*$, where A^*, B_1, and B_2 are positive parameters, and where <u>θ is a parameter that is assumed to be greater than 1</u>. In addition, B_1 and B_2 are assumed to be such that $\dot{A}$ does not change discontinuously when A reaches A^*. This requires that $B_1 a_L L A^{*\theta} = B_2 a_L L$, which is equivalent to $B_2 = B_1 A^{*\theta}$.

The initial level of knowledge, A(0), is assumed to be greater than zero and less than A^*.

a. Consider the period when A is less than A^*.

i. Define $g_A(t) \equiv \dot{A}(t)/A(t)$. What is $g_A(t)$ as a function of B_1, a_L, L, and A(t)?

ii. Find an expression for $\dot{g_A}(t)$ as a function of $g_A(t)$ and θ.

iii. Is $g_A(t)$ rising, falling, or constant over time?

b. Now consider the period when A is greater than or equal to A^*.

i. What is $\dot{A}(t)$?

ii. Is $g_A(t)$ rising, falling, or constant over time?

c. Combine your answers to (a) and (b) to:

i. Sketch the path of the growth rate of output, $\dot{Y}(t)/Y(t)$ over time.

ii. Sketch the path of the log of output, ln Y(t), over time.

9-14. Romer, Problems 3.5, 3.8, 3.14, 4.1, 4.4, 4.9.