UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2016
D. Romer

Problem Set 4 – Revised
Supersedes previous version
Due in lecture Thursday, September 29

1. a. Explain in a few sentences (with or without math) what is wrong with the following argument: "In the planner's problem in the Ramsey-Cass-Koopmans model, if capital exceeds the golden-rule level, the value of capital (that is, the amount at the margin that an increase in capital contributes to the planner's objective function) is negative. We can see this from the equation of motion for the costate variable: $\dot{\mu}(t) = \mu(t)[f'(k(t)) - (n + g)] + \beta\mu(t)$. If capital exceeds its golden-rule level, *f'(k)* is less than $n + g$, and so the contribution of capital to social welfare at t is negative."

b. Explain in one sentence what is wrong with the following argument: "The premise of the argument in part (a) makes no sense, because one of the central results of the model is that capital can never be greater than its golden-rule level."

2. In a Diamond economy with logarithmic utility, $U_t = \ln C_{1t} + [\ln C_{2,t+1}/(1 + \rho)]$, and Cobb-Douglas production, $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$, a rise in individuals' discount rate, ρ:

A. Shifts the locus showing k_{t+1} as a function of k_t down.

B. Shifts the locus showing k_{t+1} as a function of k_t up.

C. Does not affect the locus showing k_{t+1} as a function of k_t.

D. Has an ambiguous effect on the locus showing k_{t+1} as a function of k_t.

3. Romer, Problem 2.18.

4. Romer, Problem 2.20.

5. (The Diamond model with labor supply in both periods of life.) Consider the Diamond overlapping-generations model. Assume, however, that each individual supplies one unit of labor in <u>each</u> period of life. For simplicity, assume no population growth; thus total labor supply is $2L$, where L is the number of individuals born each period.

In addition, assume that there is no technological progress, and that production is Cobb-Douglas. Thus, $Y_t = BK_t^{\alpha}[2L]^{1-\alpha},\ B > 0, 0 < \alpha < 1$. Factors are paid their marginal products.

The utility function of an individual born at time t is $U_t = \ln C_{1,t} + \ln C_{2,t+1}$.

Finally, there is 100 percent depreciation, so $K_{t+1} = Y_t - [LC_{1,t} + LC_{2,t}]$.

a. Consider an individual born in period t who receives a wage of w_t in the first period of life and a wage of w_{t+1} in the second period, and who faces an interest rate of r_{t+1}. What is the individual's first-period consumption and saving as a function of w_t, w_{t+1}, and r_{t+1}?

b. What will be the wage at t as a function of K_t? What will be the interest rate at t as a function of K_t? (Hint: Don't forget that the depreciation rate is not assumed to be zero.)

c. Explain intuitively why $K_{t+1} = (w_t - C_{1,t})L$.

d. Derive an equation showing the evolution of the capital stock from one period to the next.

(NOTE: MORE ASSIGNED PROBLEMS ON NEXT PAGE)

6. (This is different from the usual type of problem.) This problem asks you to "play around" with the model in Section 2.1 of Eggertsson-Mehrotra. Specifically, choose some simplification, generalization, or variation of the assumptions of the model (or of the slightly simplified version of that model presented in lecture). Explain why you chose the change to the assumptions that you did. Then investigate how, if at all, your change affects the basic analysis and messages of the model, and discuss what you found.

Obviously, there is no right answer to this question. For example, if a seemingly small variation or generalization makes the model intractable, or if an apparent simplification does not make the model any easier to analyze or more transparent, or if an apparent generalization turns out not to be a generalization at all, that would be interesting to know from the perspective of model-building and of understanding the model.

Likewise, there is no right or wrong motivation for changing the model. Nonetheless, it is worth spending some time thinking about what change you want to make. Examples of potentially promising motivations are, "Looking at their analysis, it seemed to me that all that assumption xxxx did was to clutter up the presentation without generating any insights; I wanted to see if this was true"; or, "I can argue intuitively that the results would fall apart if I relaxed assumption yyyy; I wanted to see whether this was true."

7. (Natural resources in a model of knowledge accumulation.) Consider the following variant of the model of knowledge accumulation and growth in Section 3.2 of *Advanced Macroeconomics*. $R(t)$ denotes use of natural resources at time t, and a_R denotes the fraction of those resources that are used in the R&D sector. The rest of the notation is standard.

$$Y(t) = A(t)[(1 - a_L)L(t)]^{\beta}[(1 - a_R)R(t)]^{1-\beta}, \quad 0 < a_L < 1, 0 < a_R < 1, 0 < \beta < 1,$$

$$\dot{L}(t) = nL(t), \quad n > 0,$$

$$\dot{R}(t) = -\mu R(t), \quad \mu > 0,$$

$$\dot{A}(t) = B[a_L L(t)]^{\gamma}[a_R R(t)]^{\varphi} A(t)^{\theta}, \quad B > 0, \gamma > 0, \varphi > 0.$$

Assume $\theta < 1$. $A(0)$, $L(0)$, and $R(0)$ are all strictly positive.

a. Define $g_A(t) \equiv \dot{A}(t)/A(t)$. Derive an expression for $\dot{g}_A(t)$ in terms of $g_A(t)$ and the parameters.

b. Sketch the function you found in part (a). For what values of g_A is $\dot{g}_A = 0$? For what parameter values and/or initial conditions does g_A converge to each of these values?

c. What is the growth rate of output per person on the balanced growth path as a function of the parameter values and/or initial conditions?

(NOTE: POSTED VERSION HAS EXTRA PROBLEMS NOT TO BE HANDED IN)

EXTRA PROBLEMS (NOT TO BE HANDED IN/ANSWERS WILL GENERALLY NOT BE PROVIDED)

8. Romer, Problem 2.14.

9. Consider an economy described by the Diamond overlapping-generations model where initially k is above its balanced-growth-path level. Now suppose there is an unexpected, permanent rise in agents' discount rate, ρ.

Sketch the resulting paths of k, and what that path would have been if ρ had not changed. Explain your answer.

10. In a Diamond economy, the balanced growth path cannot be dynamically inefficient if:

A. Utility is logarithmic and production is Cobb-Douglas.
B. Individuals' discount rate (ρ) exceeds the economy's growth rate ($n + g$).
C. The initial capital stock is less than the golden rule capital stock.
D. None of the above.

11. Romer, Problem 2.19.

12. Romer, Problem 2.21.

13. Romer, Problem 2.17.

14. Romer, Problem 3.1.