UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2016
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Problem Set 2
Due in lecture Tuesday, September 13

1. In reading and lecture, we linearized the equations of motion for k and y around k* and y*. In many contexts, however, it is more helpful to work with loglinearized than with linearized systems. Thus: Linearize the equation of motion for ln k around ln k*, and simplify the resulting expression as much as possible.

2. Romer, Problem 2.3.

3. Consider an infinitely-lived household maximizing the utility function $\int_{t=0}^{\infty} e^{-\rho t} U\big(C(t)\big) dt$ subject to the usual intertemporal budget constraint. Let $r(t)$ denote the real interest rate at t and let $R(t) \equiv \int_{\tau=0}^{t} r(\tau) d\tau$. Then the Euler equation relating consumption at two dates, A and B (B > A) is:

A. $\frac{\dot{C}(A)}{C(A)} = \frac{[r(t)-\rho]}{\theta}$.

B. $\frac{\dot{C}(A)}{C(A)} = \frac{\dot{C}(B)}{C(B)}$.

C. $U'\big(C(A)\big) = \left[\frac{e^{R(B)-R(A)}}{e^{\rho(B-A)}}\right] U'\big(C(B)\big)$.

D. $U'\big(C(A)\big) = \left[\frac{[r(B)-r(A)]}{[\rho(B-A)]}\right] U'\big(C(B)\big)$.

4. (Using the calculus of variations to analyze the problem of accumulating worthless trash.) Consider an individual choosing the path of G to maximize $\int_{t=0}^{\infty} e^{-\rho t} \left[-\frac{a}{2} G(t)^2\right] dt,\ a > 0, \rho > 0$. Here $G(t)$ is the amount of garbage the individual creates at time t; for simplicity, we allow for the possibility that G can be negative. The individual's creation of garbage affects his or her stock of trash. In particular, the stock of trash, T, evolves according to $T(0) = 0$, $\dot{T}(t) = G(t)$.

a. Prove – using as little math as possible – that the utility-maximizing path is $G(t) = 0$ for all t.

b. Now, let's analyze this problem using the calculus of variations. Let G be the control variable and T the state variable, and let μ denote the costate variable. What is the current value Hamiltonian?

c. Find the conditions for optimality other than the transversality condition. Describe the paths of G that satisfy those conditions.

d. What is the transversality condition? Show that it rules out all but one of the paths you found in part (c), and that the one remaining path is the one that you showed in part (a) to be optimal: $G(t) = 0$ for all t.

e. Explain in a sentence or two why the solutions in (c) other than $G(t) = 0$ for all t look as if they are utility-maximizing if one does not consider the transversality condition, and why the transversality condition rules them out.

(NOTE: ONE MORE ASSIGNED PROBLEM ON NEXT PAGE)

5. (From the Fall 2013 final exam.) Consider an infinitely-lived household. The household's initial wealth, A(0) is zero; its labor income is constant and equal to $\bar{Y}, \bar{Y} > 0$; and the real interest rate is constant and equal to $\bar{r} > 0$. The household's flow budget constraint is therefore $\dot{A}(t) = \bar{r}A(t) + \bar{Y} - C(t)$, and, as usual, the present discounted value of the household's consumption cannot exceed the present discounted value of the its lifetime resources.

In contrast to our usual model, however, the household obtains utility not only from consumption, but also from holding wealth. Specifically, its objective function is

$$\int_{t=0}^{\infty} e^{-\rho t}[u(C(t)) + v(A(t))]dt,$$

where $u'(\bullet) > 0$, $u''(\bullet) < 0$, $v'(\bullet) > 0$, $v''(\bullet) < 0$, and $\rho > 0$.

a. For this part only, assume $\rho = \bar{r}$. Without doing any math, explain whether $C(0)$ will be less than, equal to, or greater than $\bar{Y}$, or whether it is not possible to tell.

b. What is the current value Hamiltonian?

c. Find the conditions that characterize the solution to the household's maximization problem. Use them to find an expression for $\dot{C}(t)/C(t)$ that does not involve the costate variable.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

6. Show that on the balanced growth path of the Solow model, $K/Y = s/(n + g + \delta)$.

7. Consider an economy described by the Solow model that is on its balanced growth path. Assume that the saving rate is s_0. Now suppose that from time t_0 to time t_1, the saving rate rises gradually from s_0 to s_1 (where $s_1 > s_0$), and then remains at s_1.

Sketch the resulting path over time of log output per worker. For comparison, also sketch on the same graph: (i) the path that log output per worker would have followed if the saving rate had remained at s_0; (ii) the path that log output per worker would have followed if the saving rate had jumped discontinuously from s_0 to s_1 at time t_0 (and remained at s_1).

Explain your answer.

8. Romer, Problem 1.10.

9. Romer, Problem 2.2.

10. Romer, Problem 2.4.

EXTRA EXTRA PROBLEM (NOT TO BE HANDED IN/NO ANSWER WILL BE PROVIDED)

11. Romer, Problem 1.12.