UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2016
D. Romer

Problem Set 1
Due in lecture Tuesday, September 6

Problem set ground rules:

- You may work with classmates, but: (1) You must try the problems on your own first; (2) You must write up your answers yourself, in your own words.; (3) If you work with others, you should thank them in an acknowledgment footnote at the start of your problem set.

- Looking at already written answers to the problems in any form is not permitted and would be considered academic misconduct.

1. (Hicks meets Solow.) Consider the Solow model with one change: the production function is Y = AF(K,L). All other assumptions of the model are unchanged. (The type of technical change assumed in the Solow model is known as labor-augmenting or Harrod-neutral technical change; the form of technical change assumed in this problem is called Hicks-neutral.)

a. Show what happens if we try to derive a balanced growth path like the one derived in class.

b. What can you say in the special case $F(K,L) = K^{\alpha}L^{1-\alpha}$, $0 < \alpha < 1$?

2. (This is from last year's midterm.) In the Solow model with all of our usual assumptions, except that $n + g + \delta = 0$:

A. *k* would converge to its unique balanced growth path value of 0.

B. *k* would converge to some strictly positive, finite value.

C. *k* would grow without bound.

D. It is not possible to determine the behavior of *k*.

3. Romer, Problem 1.9.

4. This question asks you to use a Solow-style model to investigate some ideas that have been discussed in the context of Thomas Piketty's recent work.

Consider an economy described by the assumptions of the Solow model, except that factors are paid their marginal products (as in the previous problem), and all labor income is consumed and all other income is saved. Thus, $C(t) = L(t)\left[\frac{\partial Y(t)}{\partial L(t)}\right]$.

a. Show that the properties of the production function imply that the capital-output ratio, *K*/Y, is rising if and only if the growth rate of *K* is greater than $n + g$ – that is, if and only if *k* is rising.

b. Assume that the initial conditions are such that $\partial Y/\partial K$ at $t = 0$ is strictly greater than $n + g + \delta$. Describe the qualitative behavior of the capital-output ratio over time. (For example, does it grow or fall without bound? Gradually approach some constant level from above or below? Something else?) Explain your reasoning.

c. Many popular summaries of Piketty's work describe his thesis as: Since the return to capital exceeds the growth rate of the economy, the capital-output ratio tends to grow without bound. By the assumptions in part (b), this economy starts in a situation where the return to capital exceeds the economy's growth rate. If you found in (b) that *K/Y* grows without bound, explain intuitively whether the driving force of this unbounded growth is that the return to capital exceeds the economy's growth rate. Alternatively, if you found in (b) that *K/Y* does not grow without bound, explain intuitively what is wrong with the statement that the return to capital exceeding the economy's growth rate tends to cause *K/Y* to grow without bound.

d. Suppose $F(\cdot)$ is Cobb-Douglas and that the initial situation is as in part (b). Describe the qualitative behavior over time of the share of *net* capital income (that is, $K(t)\left[\frac{\partial Y(t)}{\partial K(t)} - \delta\right]$) in *net* output (that is, $Y(t) - \delta K(t)$). Explain your reasoning. Is the common statement that an excess of the return to capital over the economy's growth rate causes capital's share to rise over time correct in this case?

EXTRA PROBLEMS (NOT TO BE HANDED IN; ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

5. Describe how, if at all, each of the following developments affects the break-even and actual investment lines in our basic diagram for the Solow model:

a. The rate of population growth falls.

b. The rate of technological progress rises.

c. The production function is Cobb-Douglas, $F(K,AL) = K^{\alpha}(AL)^{1-\alpha}$, and capital's share, α, rises.

d. Workers exert more effort, so that output per unit of effective labor for a given value of capital per unit of effective labor is higher than before.

6. (This is based on a problem on the 2014 midterm.) Consider the Solow model without technological progress. Suppose that, in contrast to the usual assumption in the model, population growth is higher when output per worker is higher. Specifically, assume that $n = n(y)$, with $n'(\bullet) > 0$, where y is output per worker. Assume that $n(y) + \delta > 0$ for all y.

a. Describe how, if at all, this change affects our usual diagram for the Solow model – that is, the diagram showing actual investment per worker and break-even investment per worker as functions of capital per worker.

b. Suppose the economy is on a balanced growth path. Let n^* denote the rate of population growth on this balanced growth path. Now suppose there is a permanent increase in the saving rate.

i. Does this change permanently affect the growth rate of output per worker? Why or why not? Does it permanently affect the growth rate of total output? Why or why not?

ii. Is the long-run effect of the increase in the saving rate on the level of output per worker bigger, smaller, or the same as it would be if population growth stayed constant at n^*? Explain your answer.

c. Derive an expression for the elasticity of the balanced-growth-path value of output per worker, y^*, with respect to the saving rate, s. Simplify the resulting expression as much as possible.

7. In the Solow model, a discontinuous rise in s causes $\dot{k}$ (that is, dk/dt) to:

A. Increase discontinuously.

B. Start rising at a faster rate than before, but not to increase discontinuously.

C. Increase discontinuously if the initial level of k is below its new balanced-growth-path value; decrease discontinuously if the initial level of k is above its new balanced-growth-path value; and not change if the initial level of k is equal to its new balanced-growth-path value.

D. Decrease discontinuously, but then rise at a faster rate than before.

8. Consider an economy described by the Solow model. Assume that initially capital and output per unit of effective labor are *less* than their balanced-growth-path values. Now suppose that in this situation, the saving rate rises permanently.

Sketch the resulting path of the log of output per worker and what that path would have been if the saving rate had not changed. Explain your answer.

9. (Problem 4, cont.) Consider a marginal increase in K.

a. Derive an expression (in terms of: K/Y, δ, the marginal product of capital F_K, and the elasticity of substitution between capital and effective labor in the gross production function $F(\bullet)$) that determines whether a marginal increase in K increases, reduces, or has no effect on the share of net capital income in net output.

b. Suppose the capital-output ratio is 3, $\delta = 3\%$, and the rate of return on capital $(F_K - \delta) = 5\%$. How large must the elasticity of substitution in the gross production function be for the share of net capital income in net output to rise when K rises?