University of California
Department of Economics

Economics 202A
Fall 2015
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Problem Set 12
Due in Lecture Thursday Dec. 10

1 The Optimal Exchange Rate Feedback Rule (from Obstfeld and Rogoff, 9.3)

Consider the following stochastic small open economy model in which all exogenous variables are constant except for serially uncorrelated shocks, and $p^* = i^* = \bar{y} = \bar{q} = 0$:

$$\begin{aligned} i_{t+1} &= E_t e_{t+1} - e_t \\ y_t^s &= \theta(p_t - E_{t-1} p_t) \\ y_t^d &= \delta(e_t - p_t) + \epsilon_t \\ m_t - p_t &= -\eta i_t + \phi y_t + \mu_t \end{aligned}$$

Here $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $\mu \sim \mathcal{N}(0, \sigma_\mu^2)$ are independent serially uncorrelated normally distributed shocks. The second equation above is a simple rational-expectations supply curve in which one-period price surprises can raise or lower output. The shock ϵ may be thought of as a shock to the demand for a country's goods, and μ as a shock to the demand for real money balances. We assume that the objective function of the monetary authorities is to minimize the one-period conditional variance of output $E_{t-1}[y_t^2]$.

1. First consider a fixed money supply rule under which $m_t = \bar{m}$ in all periods, and calculate $E_{t-1}[y_t^2]$ as a function of σ_ϵ^2, σ_μ^2 and the other parameters of the model. [Hint: you will find that under that policy $E_t e_s = E_t p_s = \bar{m}$ for $s > t$.]

2. Now suppose that monetary authorities fix the exchange rate at $\bar{e} = \bar{m}$ by adjusting $m_t - \bar{m}$ in response to the shocks each period as necessary to hold the exchange rate constant. They do not, however, alter the announced future path of money, which is expected to remain at $\bar{m}$ in the absence of future shocks (that is $E_t m_s = \bar{m}$ for $s > t$.) Again, calculate $E_{t-1}[y_t^2]$.

3. Show that as $\sigma_\epsilon^2/\sigma_\mu^2 \to 0$, the policy in part 2 of a fixed exchange rate is always superior to the policy in part 1 of a fixed money supply (pure float).

4. [Not graded: very hard] Suppose that instead of limiting themselves to a pure fixed rate or a pure float, the monetary authorities adopt an exchange rate feedback rule of the form $m_t - \bar{m} = -\Phi(e_t - \bar{e})$. Find the optimal value of Φ, and show that, in general, it is intermediate between 0 (pure float) and ∞ (fixed exchange rate). [Hint: this is not conceptually difficult, but there is a fair amount of algebra.]

2 Rollover Crises

Consider a small open economy with two periods, $t = 0$ and $t = 1$. In the first period, the government needs to roll-over a maturing debt D_0. The government does not have any fiscal revenues this period and must issue some new debt with *face value* D_1 (i.e D_1 is the promised repayment in period 1) at some price q_1 that we will characterize. IN period 0, the government can also implement a fiscal reform at cost e. Consumption in period $t = 0$ is: $c_0 = q_1 D_1 - D_0 - e$. c_0 must be positive.

In period 1, the government receives a primary surplus $S = \bar{S}(e)\epsilon$ where $\bar{S}(e) > 0$ is the expected level of the fiscal surplus as a function of the reform effort e and ϵ is distributed according to the cdf $G(\epsilon)$ on the interval $[\epsilon_{\min}, \epsilon_{\max}]$, with $\epsilon_{\min} > 0$ and $E[\epsilon] = 1$. The expected fiscal surplus is concave in effort: $\bar{S}'(e) > 0$ and $\bar{S}''(e) < 0$. Assume also that $\lim_0 \bar{S}'(0) = \infty$ and $\lim_\infty \bar{S}(e) = 0$.

If the government defaults in period 1, lenders can seize a fraction $0 < \eta < 1$ of the realized primary surplus $\bar{S}(e)\epsilon$. The government is strategic: it defaults if the benefit exceeds the cost. Consumption in the second period is $c_1 = \bar{S}(e)\epsilon - D_1$ in case of repayment and $c_1 = (1-\eta)\bar{S}(e)\epsilon$ in case of default.

1. Start from period 1, taking the debt level D_1 and the reform effort e as given. Show that there is a critical value of the fiscal shock, $\tilde{\epsilon}$, such that the government prefers to repay if $\epsilon \geq \tilde{\epsilon}$ and to default if $\epsilon < \tilde{\epsilon}$. Express $\tilde{\epsilon}$ as a function of D_1 and e.

2. The government borrows from risk-neutral lenders. The lenders' opportunity cost of funds is the risk free rate, normalized to zero: $r^* = 0$. Write down the arbitrage equation that must hold for the price $q_1 = q(D_1, e)$ of domestic government debt.

3. In period 0 the government can raise revenues $H(D_1, e) \equiv q(D_1, e)D_1$. Taking the effort level as given, explain how $H(D_1, e)$ varies with D_1.

4. Still taking e as given, discuss whether there are situations of insolvency (where the government cannot repay its debt in period 0) and/or illiquidity (where the government may be unable to repay its debt in period 1). Is there a 'safe region' where the government faces no risk of crisis and if so why or why not? Are there policies that would eliminate liquidity crises in this model?

5. Commitment. Suppose that the government can simultaneously choose its debt level D_1 and commit to a reform effort e to maximize $U = c_0 + E[c_1]$. Characterize the optimal effort level e^*. What can you say about the optimal debt level D_1? Why?

6. No-commitment. Suppose now that the government gets to choose the effort level e *after* the debt level D_1 has been issued at a price q_1.

   - Characterize the optimal effort level e^{**} taking as given D_1 and q_1. Does it differ from e^*? Why?
   - Given your answer to the previous question, what can you say about the rollover problem, i.e. how much debt D_1^{**} the country will choose to issue at $t = 0$?