

Problem Set 10
Due in Lecture Tuesday Nov. 17

1 Romer 9.6

2 Romer 9.8

3 Imperfect Competition and Investment

This question is a former final exam question. Special ground rules: (1) First try it under exam conditions - no readings, no notes, no help. (2) then you may look at your notes and readings but not consult with others; (3) you may then consult with others; (4) as always, you may not look at past years problem set solutions (or past years midterm and final exam solutions).

A firm has a production function $Y_t = K_t^\alpha L_t^{1-\alpha}$ where $0 < \alpha < 1$ where K_t denotes the firm's capital and L_t its labor. The firm is a monopolist and faces a demand function for its good given by $p_t = p_0 Y_t^{-\eta}$ where $p_0 > 0$ and $0 < \eta < 1$, in terms of some numeraire. The wage (in terms of the same numeraire) is constant at w , and taken as given by the firm.

1. At time t , given a pre-determined stock of capital K_t , write the optimization program that the firm solves and show that the firm's flow profits can be expressed as $\Pi(K_t) = \pi K_t^\gamma$ for some constants π and $0 < \gamma < 1$. You will express γ in terms of α and η .
2. Time is continuous. The price of capital goods is assumed constant and equal to 1 (in terms of the numeraire). The firm faces adjustment costs of capital per unit of investment $C(I) = (a/2)I$ (i.e. total investment costs are $I(1 + C(I))$). The firm starts at time 0 with a capital stock K_0 . The real interest rate at which it discounts future profits is constant and equal to r and the depreciation rate is δ .
 - (a) write down the maximization problem that the firm solves
 - (b) Write the Hamiltonian and find the conditions that characterize the solution to the firm's maximization problem
 - (c) Suppose that at time 0 the firm is at its long run capital stock and that the goods' market becomes unexpectedly and permanently perfectly competitive at price p_0 . Explain intuitively how this affects the $\dot{q} = 0$ locus, the $\dot{K} = 0$ locus, the saddle path and the values of q and K at time 0.

4 External Adjustment Costs

This question consider what happens when firms faces *external* adjustment costs. Assume that there is continuum of firms of mass 1, each solving an identical problem. We will denote firm level variable with lower cases: k_t for capital and i_t for investment at time t . Aggregate variables will be denoted with uppercase variables: K_t and I_t for aggregate capital stock and aggregate investment at time t .

Assume that there are no internal adjustment costs (i.e. $C(i, k) = 0$) but that the price of investment goods is a function of aggregate investment: $p_K(I)$ with $p'_K(I) > 0$. Denote $\Pi(k_t)$ the profits of a firm operating with capital stock k_t at time t , $\delta > 0$ the depreciation rate and $r > 0$ the real risk-free rate.

- Write the problem that the firm solves.
- Write the present value Hamiltonian and the first-order conditions for the firm problem, taking as given aggregate variables (K and I). What does this imply for Tobin's marginal q defined as the marginal value of an extra unit of capital relative to the price of investment goods? Does it vary over time? Does the firm satisfy the neoclassical 'user-cost' rule for investment we derived in class?
- Recognizing that in equilibrium aggregate capital and aggregate investment satisfy $K = k$ and $I = i$, characterize the dynamics of the economy in a (I, K) space. Describe whether the economy exhibits saddle-path dynamics.
- Assume now that there is only one large firm in the economy, acting as a monopsonist on the market for investment goods: it recognizes that if it invests I , it faces a price schedule $p_K(I)$. Assume that the price of investment goods has a constant elasticity, that is $p_K(I) = pI^\eta$ for $\eta > 0$ and $p > 0$. Solve the firm's problem and explain how aggregate dynamics differ from the case of perfect competition.
- Goolsbee (1998) considers the effect of a an investment tax credit. This can be modeled as a reduction in the price of investment goods from $p_K(I)$ to $(1 - \tau)p_K(I)$ where $\tau > 0$ is the investment tax credit. Suppose the tax credit is unanticipated and permanent. Describe what happens to investment, capital and the price of investment goods over time in the competitive case when $p_K(I) = pI^\eta$. Explain how your answer may vary with the price elasticity of investment goods η .