UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2015
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Problem Set 5
Due in lecture Tuesday, October 6

1. Consider an economy described by: $\dot{B}(t) = bB(t)$, $\dot{D}(t) = d[cB(t)]^{\omega}D(t)^{\mu}$, $J(t) = [(1-c)B(t)]D(t)$, with $b > 0$, $d > 0$, $0 < c < 1$, $\omega > 0$, $B(0) > 0$, and $D(0) > 0$. This economy will converge to a balanced growth path if and only if:

A. $\mu < 1$.
B. $\mu \leq 1$.
C. $\omega < 1$.
D. $\omega \leq 1$.

2. Knowledge accumulation may vary in a complicated way over time. This problem asks you to investigate one way that this might occur.

For simplicity, population is constant. Output at time t is given by $Y(t) = (1 - a_L)A(t)L$, where Y is output, a_L is the fraction of the population that is engaged in producing knowledge, A is knowledge, and L is population.

Knowledge accumulation is given by the function: $\dot{A}(t) = B_1 a_L L A(t)^{\theta}$ if $A < A^*$, $\dot{A}(t) = B_2 a_L L$ if $A > A^*$, where A^*, B_1, and B_2 are positive parameters, and where θ is a parameter that is assumed to be greater than 1. In addition, B_1 and B_2 are assumed to be such that $\dot{A}$ does not change discontinuously when A reaches A^*. This requires that $B_1 a_L L A^{*\theta} = B_2 a_L L$, which is equivalent to $B_2 = B_1 A^{*\theta}$.

The initial level of knowledge, A(0), is assumed to be greater than zero and less than A^*.

a. Consider the period when A is less than A^*.
  i. Define $g_A(t) \equiv \dot{A}(t)/A(t)$. What is $g_A(t)$ as a function of B_1, a_L, L, and A(t)?
  ii. Find an expression for $\dot{g}_A(t)$ as a function of $g_A(t)$ and θ.
  iii. Is $g_A(t)$ rising, falling, or constant over time?
b. Now consider the period when A is greater than or equal to A^*.
  i. What is $\dot{A}(t)$?
  ii. Is $g_A(t)$ rising, falling, or constant over time?
c. Combine your answers to (a) and (b) to:
  i. Sketch the path of the growth rate of output, $\dot{Y}(t)/Y(t)$ over time.
  ii. Sketch the path of the log of output, ln Y(t), over time.

3. (Natural resources in a model of knowledge accumulation.) Consider the following variant of the model of knowledge accumulation and growth in Section 3.2 of *Advanced Macroeconomics*. R(t) denotes use of natural resources at time t, and a_R denotes the fraction of those resources that are used in the R&D sector. The rest of the notation is standard.

$$Y(t) = A(t)[(1-a_L)L(t)]^{\beta}[(1-a_R)R(t)]^{1-\beta}, \quad 0 < a_L < 1, 0 < a_R < 1, 0 < \beta < 1,$$

$$\dot{L}(t) = nL(t), \quad n > 0,$$

$$\dot{R}(t) = -\mu R(t), \quad \mu > 0,$$

$$\dot{A}(t) = B[a_L L(t)]^{\gamma}[a_R R(t)]^{\varphi} A(t)^{\theta}, \quad B > 0, \gamma > 0, \varphi > 0.$$

Assume $\theta < 1$. A(0), L(0), and R(0) are all strictly positive.

a. Define $g_A(t) \equiv \dot{A}(t)/A(t)$. Derive an expression for $\dot{g_A}(t)$ in terms of $g_A(t)$ and the parameters.

b. Sketch the function you found in part (a). For what values of g_A is $\dot{g_A} = 0$? For what parameter values and/or initial conditions does g_A converge to each of these values?

c. What is the growth rate of output per person on the balanced growth path as a function of the parameter values and/or initial conditions?

4. Romer, Problem 3.8.

5. In the P. Romer model of endogenous technological change, the condition for equilibrium in the allocation of workers between R&D and goods production at time *t* is:

A. The wages in the two sectors at time *t* are equal.

B. The present value of the revenues from an idea created at time *t* equals the wage in the goods-producing sector at time *t*.

C. The marginal product of an idea in creating new ideas equals its marginal product in goods production.

D. The price of using an idea equals $\eta/(\eta - 1)$ times the cost of producing the idea, where η is the elasticity of demand for the input using a given idea.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

6. Romer, Problem 3.1.

7. Romer, Problem 3.5.

8. In models where the allocation of resources to R&D is determined by market forces, the inputs that embody different ideas are typically modeled as:

A. Supplied in exogenously determined amounts.

B. Public goods.

C. Perfect substitutes for one another.

D. Imperfect substitutes for one another.

9. Romer, Problem 3.9.

10. Consider the version of the Paul Romer model presented in Section 3.5 of *Advanced Macroeconomics*. For simplicity, neglect the constraint that L_A cannot be negative. Set up the problem of choosing the path of $L_A(t)$ to maximize the lifetime utility of the representative household. What is the control variable? What is the state variable? What is the Hamiltonian? Find the conditions that characterize the optimum. Is there an allocation where $L_A(t)$ is constant that satisfies those conditions? If so, what is the constant value of L_A? If not, why not?

11. Romer, Problem 3.14.