UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2015
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Problem Set 4
Due in lecture Tuesday, September 29

1. (Using the calculus of variations to analyze the problem of accumulating worthless trash.) Consider an individual choosing the path of *G* to maximize $\int_{t=0}^{\infty} e^{-\rho t}\left[-\frac{a}{2}G(t)^2\right]dt,\ a > 0, \rho > 0$. Here *G(t)* is the amount of garbage the individual creates at time *t*; for simplicity, we allow for the possibility that *G* can be negative. The individual's creation of garbage affects his or her stock of trash. In particular, the stock of trash, *T*, evolves according to $T(0) = 0$, $\dot{T}(t) = G(t)$.

a. Prove – using as little math as possible – that the utility-maximizing path is $G(t) = 0$ for all *t*.

b. Now, let's analyze this problem using the calculus of variations. Let *G* be the control variable and *T* the state variable, and let μ denote the costate variable. What is the current value Hamiltonian?

c. Find the conditions for optimality other than the transversality condition. Describe the paths of *G* that satisfy those conditions.

d. What is the transversality condition? Show that it rules out all but one of the paths you found in part (c), and that the one remaining path is the one that you showed in part (a) to be optimal: $G(t) = 0$ for all *t*.

e. Explain in a sentence or two why the solutions in (c) other than $G(t) = 0$ for all *t* look as if they are utility-maximizing if one does not consider the transversality condition, and why the transversality condition rules them out.

2. a. Explain in a few sentences (with or without math) what is wrong with the following argument: "In the planner's problem in the Ramsey-Cass-Koopmans model, if capital exceeds the golden-rule level, the value of capital (that is, the amount at the margin that an increase in capital contributes to the planner's objective function) is negative. We can see this from the equation of motion for the costate variable: $\dot{\mu}(t) = \mu(t)[f'(k(t)) - (n+g)] + \beta\mu(t)$. If capital exceeds its golden-rule level, *f'(k)* is less than $n + g$, and so the contribution of capital to social welfare at *t* is negative."

b. Explain in one sentence what is wrong with the following argument: "The premise of the argument in part (a) makes no sense, because one of the central results of the model is that capital can never be greater than its golden-rule level."

3. In a Diamond economy with logarithmic utility, $U_t = \ln C_{1t} + [\ln C_{2,t+1} / (1+\rho)]$, and Cobb-Douglas production, $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$, a rise in individuals' discount rate, ρ:

A. Shifts the locus showing k_{t+1} as a function of k_t down.
B. Shifts the locus showing k_{t+1} as a function of k_t up.
C. Does not affect the locus showing k_{t+1} as a function of k_t.
D. Has an ambiguous effect on the locus showing k_{t+1} as a function of k_t.

4. (The Diamond model with labor supply in both periods of life.) Consider the Diamond overlapping-generations model. Assume, however, that each individual supplies one unit of labor in <u>each</u> period of life. For simplicity, assume no population growth; thus total labor supply is 2L, where L is the number of individuals born each period.

In addition, assume that there is no technological progress, and that production is Cobb-Douglas. Thus, $Y_t = BK_t^{\alpha}[2L]^{1-\alpha},\ B > 0, 0 < \alpha < 1$. Factors are paid their marginal products.

The utility function of an individual born at time t is $U_t = \ln C_{1,t} + \ln C_{2,t+1}$.

Finally, there is 100 percent depreciation, so $K_{t+1} = Y_t - [LC_{1,t} + LC_{2,t}]$.

a. Consider an individual born in period t who receives a wage of w_t in the first period of life and a wage of w_{t+1} in the second period, and who faces an interest rate of r_{t+1}. What is the individual's first-period consumption and saving as a function of w_t, w_{t+1}, and r_{t+1}?

b. What will be the wage at t as a function of K_t? What will be the interest rate at t as a function of K_t? (Hint: Don't forget that the depreciation rate is not assumed to be zero.)

c. Explain intuitively why $K_{t+1} = (w_t - C_{1,t})L$.

d. Derive an equation showing the evolution of the capital stock from one period to the next.

5. Romer, Problem 2.17.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

6. Romer, Problem 2.14.

7. In a Diamond economy, the balanced growth path <u>cannot</u> be dynamically inefficient if:

A. Utility is logarithmic and production is Cobb-Douglas.
B. Individuals' discount rate (ρ) exceeds the economy's growth rate (n + g).
C. The initial capital stock is less than the golden rule capital stock.
D. None of the above.

8. Romer, Problem 2.18.

9. Romer, Problem 2.20.