UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2015
P. Gourinchas/D. Romer

Problem Set 2
Due in lecture Tuesday, September 15

1. Saving rates may be higher at higher levels of income. This problem asks you to investigate the consequences of this possibility for economic growth.

Consider the Solow model without technological progress. For simplicity, assume that A is one, so that y and k are income per worker and capital per worker.

Now suppose that, in contrast to our usual assumptions:

- The saving rate is zero if income per worker is less than some critical level, $f(\tilde{k})$.
- The saving rate is s (where $s > 0$) if income per worker exceeds $f(\tilde{k})$.

Finally, assume that $sf(\tilde{k})$ is greater than $(n + \delta)\tilde{k}$.

A. Describe how, if at all, this change affects our usual diagram for the Solow model – that is, the diagram showing actual investment per worker and break-even investment per worker as functions of capital per worker.

B. Describe the behavior of output per worker over time if:

i. The initial level of capital per worker, $k(0)$, is between 0 and $\tilde{k}$.
ii. The initial level of capital per worker, $k(0)$, is slightly greater than $\tilde{k}$.

2. In reading and lecture, we linearized the equations of motion for k and y around k^* and y^*. In many contexts, however, it is more helpful to work with loglinearized than with linearized systems. Thus: Linearize the equation of motion for $\ln k$ around $\ln k^*$, and simplify the resulting expression as much as possible.

3. Romer, Problem 2.3.

4. Consider an infinitely-lived household maximizing the utility function $\int_{t=0}^{\infty} e^{-\rho t} U\big(C(t)\big)dt$ subject to the usual intertemporal budget constraint. Let $r(t)$ denote the real interest rate at t and let $R(t) \equiv \int_{\tau=0}^{t} r(\tau)d\tau$. Then the Euler equation relating consumption at two dates, A and B (B > A) is:

A. $\frac{\dot{C}(A)}{C(A)} = \frac{[r(t)-\rho]}{\theta}$.
B. $\frac{\dot{C}(A)}{C(A)} = \frac{\dot{C}(B)}{C(B)}$.
C. $U'\big(C(A)\big) = \left[\frac{e^{R(B)-R(A)}}{e^{\rho(B-A)}}\right] U'\big(C(B)\big)$.
D. $U'\big(C(A)\big) = \left[\frac{[r(B)-r(A)]}{[\rho(B-A)]}\right] U'\big(C(B)\big)$.

5. (From the Fall 2013 final exam.) Consider an infinitely-lived household. The household's initial wealth, A(0) is zero; its labor income is constant and equal to $\bar{Y}, \bar{Y} > 0$; and the real interest rate is constant and equal to $\bar{r} > 0$. The household's flow budget constraint is therefore $\dot{A}(t) = \bar{r}A(t) + \bar{Y} - C(t)$, and, as usual, the present discounted value of the household's consumption cannot exceed the present discounted value of the its lifetime resources.

In contrast to our usual model, however, the household obtains utility not only from consumption, but also from holding wealth. Specifically, its objective function is

$$\int_{t=0}^{\infty} e^{-\rho t}[u\big(C(t)\big) + v\big(A(t)\big)]dt,$$

where $u'(\bullet) > 0$, $u''(\bullet) < 0$, $v'(\bullet) > 0$, $v''(\bullet) < 0$, and $\rho > 0$.

A. For this part only, assume $\rho = \bar{r}$. Without doing any math, explain whether $C(0)$ will be less than, equal to, or greater than $\bar{Y}$, or whether it is not possible to tell.

B. What is the present value Hamiltonian?

C. Find the conditions that characterize the solution to the household's maximization problem.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

6. Show that on the balanced growth path of the Solow model, K/Y = s/(n + g + δ).

7. Consider an economy described by the Solow model that is on its balanced growth path. Assume that the saving rate is s_0. Now suppose that from time t_0 to time t_1, the saving rate rises gradually from s_0 to s_1 (where $s_1 > s_0$), and then remains at s_1.

Sketch the resulting path over time of log output per worker. For comparison, also sketch on the same graph: (i) the path that log output per worker would have followed if the saving rate had remained at s_0; (ii) the path that log output per worker would have followed if the saving rate had jumped discontinuously from s_0 to s_1 at time t_0 (and remained at s_1).

Explain your answer.

8. (From last year's midterm.) Consider the planner's problem in the Ramsey model, with one change: the discount rate of the representative household is not constant. Specifically, the household's lifetime utility is given by:

$$\int_{t=0}^{\infty} e^{-D(t)} u(C(t))dt, \quad u'(\bullet) > 0, \quad u''(\bullet) < 0,$$

where $D(t) \equiv \int_{\tau=0}^{t} \rho(\tau)d\tau$.

For simplicity, assume that population growth and technological progress are both zero, and normalize the number of households to 1. As usual in the Ramsey model, the economy has an initial capital stock of $K(0) > 0$, K evolves according to $\dot{K}(t) = Y(t) - C(t)$, and $Y(t) = F\big(K(t), L(t)\big)$, where $F(\bullet)$ has the usual properties.

This problem asks you to use optimal control to derive the Euler equation for consumption. Specifically:

A. Set up the Hamiltonian.

B. Find the conditions for optimality.

C. Use your results to find an expression for $\dot{C}(t)/C(t)$.

(Hint: Consider the special case of $\rho(t) = \bar{\rho}$ for all t. If your answer to (C) doesn't simplify to our usual expression for $\dot{C}(t)/C(t)$, this suggests that you've gone astray somewhere. If you have, you might want to redo (a) and (b) using the present-value Hamiltonian rather than the current-value Hamiltonian – it may be easier to see how to proceed with that approach.)

9. Romer, Problem 1.10.

10. Romer, Problem 2.2.

11. Romer, Problem 2.4.

EXTRA EXTRA PROBLEM (NOT TO BE HANDED IN/NO ANSWER WILL BE PROVIDED)

12. Romer, Problem 1.12.