Econ 101A – Solutions for Final exam - Fall 2006

Problem 1. Shorter problems. (35 points) Solve the following shorter problems.

1. Consider the following (simultaneous) game of chicken. This is a game in which two players drive cars at each other. The first to swerve away and slow down loses and is humiliated as the "chicken"; if neither player swerves, the result is a potentially fatal head-on collision. The principle of the game is to create pressure until one person backs down. Call s the probability that player 1 swerves, $1-s$ the probability that player 1 drives straight, S the probability that Player 2 swerves, and $1-S$ the probability that Player 2 drives straight. Compute all the pure-strategy and mixed strategy equilibria. (20 points)

1\2	Swerving	Driving Straight
Swerving	$0,0$	$-1,1$
Driving Straight	$1,-1$	$-10,-10$

2. Now assume that the game is played sequentially, with player 1 moving first and deciding whether to Swerve or Drive Straight, and player 2 moving second after observing what player 1 did, and deciding also whether to Swerve or Drive Straight. (I know, this makes for a boring game of chicken) Write the decision tree for this game, and find all the pure-strategy Subgame-Perfect Equilibria. In particular, write down the Subgame-Perfect Equilibrium **strategies** of this dynamic game. How do these equilibria compare to the equilibria of the static game? (15 points)

Solution of Problem 1.

1. The pure strategy Nash equilibria can be found in the matrix once we underline the best responses for each player:

1\2	Swerving	Driving Straight
Swerving	$0,0$	$\underline{-1},\underline{1}$
Driving Straight	$\underline{1},\underline{-1}$	$-10,-10$

The equilibria therefore are $(s_1^*, s_2^*) = (D,S)$ and $(s_1^*, s_2^*) = (S,D)$. (One driver swerves, the other does not) To find the mixed strategy equilibria, we compute for each player the expected utility as a function of what the other player does. We start with player 1. Player 1 prefers S to D if

$$Su_1(S,S) + (1-S)\,u_1(S,D) \geq Su_1(D,S) + (1-S)\,u_1(D,D)$$

or

$$0S - (1-S) \geq S - 10\,(1-S)$$

or

$$-1 + S \geq 11S - 10$$

or

$$S \leq 9/10.$$

Therefore, the Best Response correspondence for player 1 is

$$BR_1^*(S) = \begin{cases} s = 0 & \text{if} \quad S > 9/10; \\ \text{any } s \in [0,1] & \text{if} \quad S = 9/10; \\ s = 1 & \text{if} \quad S < 9/10. \end{cases}$$

The best-response for Player 2 is symmetric (the game is symmetric). Hence,

$$BR_2^*(s) = \begin{cases} S = 0 & \text{if} \quad s > 9/10; \\ \text{any } S \in [0,1] & \text{if} \quad s = 9/10; \\ S = 1 & \text{if} \quad s < 9/10. \end{cases}$$

Plotting the two Best Response correspondences, we see that the three points that are on the Best Response correspondences of both players are $(\sigma_1^*, \sigma_2^*) = (s = 1, S = 0)$, $(s = 1, S = 0)$, and $(s = 9/10, S = 9/10)$. The first two are the pure-strategy equilibria we had identified before, the other one is the additional equilibrium in mixed strategies. In the mixed strategy equilibrium, therefore, the probability of an accident is $1/10 * 1/10 = 1/100$.

2. We apply sub-game perfect equilibrium by working backwards. First, we define the strategies of the players. Players 1's strategy set is $\{S, D\}$. Player 2's strategies are more complicated, they are defined as the combination of what Player 2 does in each decision node, that is, as a function of what Player 1 did in the past. Hence, Player 2's strategies are $S_2 = \{\{S, S\}, \{S, D\}, \{D, S\}, \{D, D\}\}$, where the first strategy refers to what player 2 does if Player 1 swerved, and the second refers to what player 2 does if player 1 drove straight. By inspection of the tree, it is clear that if Player 1 swerves, Player 2 will drive straight, and if Player 1 drives straight, Player 2 will swerve. That is, the optimal strategy for player 2 is $s_2^* = \{D, S\}$. Knowing this, Player 1 prefers to drive straight (payoff of 1) than to swerve (payoff of -1). The sub-game perfect equilibrium strategies therefore, are $s_1^* = D, s_2^* = \{D, S\}$. The dynamic game, therefore, selects one of the two equilibria, the one that benefits Player 1.

Problem 2. Consumption-Savings Problem with Habit Formation (35 points) In this exercise, we consider the choice of consumption over time. We assume two periods, $t = 1$ and $t = 2$. Kim receives no income in period 1 and earns income M in period 2. She can borrow at per-period interest r on each dollar. The peculiarity of this exercise is that Kim's preferences are characterized by habit formation: the more she consumes in the first period, the less she gets utility from consumption in the second period. (This is similar to the addiction problem that you solved in the problem sets). More precisely, Kim has utility function

$$u(c_1, c_2) = u(c_1) + u(c_2 - \gamma c_1)$$

with $0 \leq \gamma \leq 1$, and $u(x)$ with the usual concavity assumptions, $u' > 0$ and $u'' < 0$. (We assume that $\delta = 0$)

1. Derive the intertemporal budget constraint
$$c_1 + \frac{c_2}{1+r} = \frac{M}{1+r}.$$
(5 points)

2. Write down the utility maximization problem by substituting in the expression for c_2 from the budget constraint (4 points)

3. Derive the first-order condition for c_1^*. (5 point)

4. Check the second-order condition for c_1^*. (5 point)

5. Use the implicit function theorem to compute $\partial c_1^* / \partial \gamma$. Interpret the sign. (8 points)

6. Under the assumption $u(x) = \log(x)$, solve for c_1^* and c_2^*. In particular, assume $r = 0$ for simplicity and interpret the solution for the cases $\gamma = 0$ and $\gamma = 1$. (8 points)

Solution of Problem 2.

1. In period 2, player 2 consumes $c_2 = M + S_2$, where S_2 is defined as the savings/debt carried over to period 2 from period 1. Since $S_2 = (-c_1) * (1 + r)$, [remember, there are no earnings in period 1] we have
$$c_2 = M - c_1(1 + r)$$
or, dividing by $(1 + r)$,
$$c_1 + \frac{c_2}{1+r} = \frac{M}{1+r}.$$

2. The utility maximization problem can be written as
$$\max_{c_1, c_2} u(c_1, c_2) = u(c_1) + u(c_2 - \gamma c_1)$$
$$s.t.\ c_1 + \frac{c_2}{1+r} \leq \frac{M}{1+r}.$$
Substituting in, we obtain
$$\max_{c_1} u(c_1, c_2) = u(c_1) + u(M - c_1(1 + r) - \gamma c_1)$$

3. The first-order-condition with respect to c_1 is
$$u'(c_1) - (1 + r + \gamma) u'(M - c_1(1 + r) - \gamma c_1) = 0.$$

4. The second order condition is

$$u''(c_1) + (1+r+\gamma)^2 u''(M - c_1(1+r) - \gamma c_1) < 0$$

which is satisfied given the assumptions of $u''(x) < 0$ for all x.

5. We apply the implicit function theorem to the first-order condition (which is an implicit function) as follows:

$$\frac{\partial c_1^*}{\partial \gamma} = -\frac{\frac{\partial (f.o.c.)}{\partial \gamma}}{\frac{\partial (f.o.c.)}{\partial c_1^*}} = -\frac{-u'(M - c_1(1+r) - \gamma c_1) + (1+r+\gamma) c_1 u''(M - c_1(1+r) - \gamma c_1)}{u''(c_1) + (1+r+\gamma)^2 u''(M - c_1(1+r) - \gamma c_1)}$$

The numerator is the sum of two negative terms, and the denominator is also negative, which we know from the second-order conditions. Thus, taking into account the negative sign outside, we can conclude that $\frac{\partial c_1^*}{\partial \gamma}$ is going to be negative. The more habit-forming the good is, the less I consume in the first period in order to avoid developing a habit. In other words, the individuals is aware of a negative effect of consuming too much in period 1 and exercizes restraint.

6. If we assume $u(c) = \log(c)$, we can re-write the f.o.c. as:

$$\frac{1}{c_1} - \frac{(1+r+\gamma)}{M - c_1(1+r+\gamma)} = 0$$

or

$$M - c_1(1+r+\gamma) = (1+r+\gamma) c_1$$

or

$$c_1^* = \frac{M}{2(1+r+\gamma)}.$$

The solution for c_2^* follows from the budget constraint:

$$c_2 = M - c_1^*(1+r) = M - \frac{(1+r)M}{2(1+r+\gamma)} = M\frac{1+r+2\gamma}{2(1+r+\gamma)}$$

which is decreasing in γ, as can be seen most easily from the next-to-last expression. Under the assumption $r = 0$, the case $\gamma = 0$ (no habit formation) yields $c_1^* = M/2 = c_2^*$, that is, we get the usual consumption smoothing result. In the case $\gamma = 1$ (full habit formation), we get $c_1^* = M/4$ and $c_2^* = 3M/4$, that is, the consumer consumes three times as much in the second period than in the first period. Again, this is because consuming much in the first period reduces the utility later. On the other hand, the consumer does not want to consume 0 in the first period, since that would give utility $-\infty$ in the first period.

Problem 3. Price Discrimination. (48 points) Consider a monopolistic firm that sells drugs in two markets, Europe and US. In Europe, inverse demand is given by $p(x_E) = 8 - 2x_E$, whereas in the US it is given by $p(x_{US}) = 10 - x_{US}$. The cost function of the monopolist is $c(x_E + x_{US}) = x_E + x_{US}$, that is, the marginal cost of production is constant and equal to 1.

1. Which market has a higher willingness to pay (that is, consumers are willing to pay more for a given quantity)? (5 points)
2. First, assume that the monopolist can price discriminate between the two markets, that is, can charge different prices in the two markets. Set up the maximization problem of the monopolist. (5 points)
3. Solve the problem. Compute the profit-maximizing choice of x_E^D and x_{US}^D, as well as the equilibrium prices p_E^D and p_{US}^D. (8 points)
4. Compare the quantities and prices across the two markets. Interpret the results in light of what you discussed in point 1. (5 points)
5. Now, assume that new legislation makes it illegal to price-discriminate between the two markets. That is, the monopolist must charge the same price $p_{US}^M = p_E^M = p^M$. Set up the new maximization problem for the firm. (Be careful how you set this problem up, this is not a trivial step, it may be useful to write the maximization as a function of p and X, the total production) (10 points)
6. Compute the profit-maximizing choice of the total quantity produced X^M, the price p^M, and the quantities sold in each market x_E^M and x_{US}^M. [Help: You should get $X^M = 25/4$] (10 points)
7. Compare the price p^M with the prices p_E^D and p_{US}^D with discrimination. Do a similar comparison for quantities produces with discrimination. Discuss. (5 points)

Solution of Problem 3.

1. Europe has a higher willingness to pay than the United States for a given quantity. $10 - x \geq 8 - 2x$ if $x \geq -2$, that is, always given that x is positive.
2. The firms maximizes
$$\max_{x_E, x_{US}} (8 - 2x_E)\, x_E + (10 - x_{US})\, x_{US} - (x_E + x_{US})$$
3. The first order conditions are
$$\begin{aligned} 8 - 4x_E - 1 &= 0 \\ 10 - 2x_{US} - 1 &= 0 \end{aligned}$$
which leads to $x_E^* = 7/4$ and $x_{US}^* = 9/2$. This implies $p_E^D = 8 - 2\,(7/4) = 9/2$ and $p_{US}^D = 10 - (9/2) = 11/2$.
4. Hence, the US market which is more willing to pay, pays more for the drugs ($p_{US}^* > p_E^*$) and, despite the higher price, purchases more drugs ($x_{US}^* < x_E^*$).
5. To do this, you cannot just write the problem as before because you need to impose $p_E = p_{US}$. Hence, you need to transform things so that you can write everything in terms of the total demand X and the price p. Invert the demand functions to get $x_E(p) = 4 - p/2$ and $x_{US}(p) = 10 - p$, and then add them up to obtain $X(p) = 4 - p/2 + 10 - p = 14 - 3p/2$. We can then invert it to get $p(X) = 28/3 - 2X/3$. The maximization is
$$\max_{x} (28/3 - 2X/3)\, X - X.$$

6. This leads to the first-order condition

$$\frac{28}{3} - \frac{4}{3}X - 1 = 0,$$

or $X^M = 25/4$, which implies $p^M = 28/3 - 2/3 * (25/4) = 28/3 - 25/6 = 31/6$. The quantities sold in each market are $x_E^M(p) = 4 - p/2 = 4 - 31/12 = 17/12$ and $x_{US}^M(p) = 10 - 31/6 = 29/6$.

7. The price charged in the case of no-discrimination is in between the price that in the case of discrimination were charged in Europe and US. The company is finding a mid-point. As for quantities, the company produces less in Europe now and more in the US. By virtue of charging a mid-price, the company ends up selling more in the US (since the price is now lower than before), but less in Europe (since the price is now higher there).

[**New Blue Book**]

Problem 4. Profit Maximization with Discrete Increments (118 points). In class, we studied all sorts of variants of profit maximization with continuous demand and supply functions. In this problem we study the case where the demand and supply function are discrete. You will not take any derivatives in this problem. Consumers in 101World like kiwis, but in different ways: 100 consumers value kiwis at \$5 a piece, 10 consumers value them at \$3 a piece, 90 consumers value them at \$2 a piece, and 100 consumers value them at \$1 a piece. No consumer wants more than 1 kiwi, that is, they value the second kiwi at \$0. *Assume that consumers purchase kiwis even if their valuation is exactly equal to the price of a kiwi (in which case they are indifferent).* For example, if the price is \$2, all the 90 consumers of the third type still purchase kiwis.

1. **Demand.** Plot the market demand for kiwis in the space (quantity of kiwis, price of a piece of kiwi). Put the price in the y axis. (4 points)

2. **Perfect Competition in the Short-Run.** Consider now farms producing kiwis in a perfectly competitive market. Each farm can produce 10 kiwis at the cost of \$1 each and additional kiwis at \$2. (To produce additional kiwis one needs to grow them on less productive land, which is more costly). Compute and plot the total cost, the average cost, the marginal cost for each firm. (4 points)

3. Derive the supply function for each firm. Remember, the supply function is a correspondence $y^*(p)$ from prices to quantity produced. (6 points)

4. Still under perfect competition: Assume that there are 10 firms in the market. (That is, we are in the short-run with a fixed number of firms) Compute and plot the industry supply function. (4 points)

5. Find the market equilibrium for price p^*_{PC} and total quantity produced under perfect competition Q^*_{PC} with 10 firms producing as the levels that equate demand and supply. (As I wrote above, assume that all consumers that are indifferent between purchasing and not purchasing actually purchase) (4 points)

6. Compute the firm surplus (that is, the profit) for each of the firms and the consumer surplus for each type of consumer. Compute the total surplus, defined as the sum of the surpluses by all firms and consumers. (6 points)

7. Do firms make zero profits? If so, is it surprising? If not, is it surprising? (6 points)

8. **Perfect Competition in the Long-Run.** Now, still assume perfect competition, but allow for free entry. (That is, we are in the long-run) That is, more firms with cost function of the type above will enter the market as long as there are positive profits. How many firms will enter at a minimum ? (additional firms may enter beyond this number) Plot the industry supply function in this case and determine the equilibrium price and quantity produced. (8 points)

9. Compute the firm surplus (that is, the profit) for each of the firms and the consumer surplus for each type of consumer. Compute the total surplus, defined as the sum of the surpluses by all firms and consumers. How do these differ from the case of perfect competition with fixed number of firms? Discuss. (6 points)

10. **Monopoly.** In the next year, expectations are that one big company will buy out all ten farms and act as a monopolist. Hence, this firm will be able to produce up to 100 kiwis for \$1 each and any additional kiwi for \$2 each. (That is, the costs for this firm are the sum of the costs for the 10 individual firms) Determine the profit-maximizing quantity q^*_M and price p^*_M produced by the monopoly. (The firm charges the same price to all consumers) (8 points)

11. Compute the firm surplus (that is, the profit) and the consumer surplus for each type of consumer. Compute the total surplus, defined as the sum of the surpluses by the firm and all the consumers. How do these differ from the previous cases you considered? Discuss. (6 points)

12. **Monopoly with Perfect Price Discrimination.** Assume now that a single monopolist can charge different prices to different consumers. What will the prices and quantities in equilibrium be now? (6 points)

13. Compute the firm surplus (that is, the profit) and the consumer surplus for each type of consumer. Compute the total surplus, defined as the sum of the surpluses by the firm and all the consumers. (4 points)

14. Focus on the total surplus and compare the case of perfect competition in the short-run and the two monopoly cases. Discuss and relate to deadweight loss. (6 points)

15. **Bertrand Duopoly.** Based on the analysis above, the anti-trust authority outlaws the buy-out plans that would lead to monopoly. The authority instead will allow two firms to take over half the market each. That is, each firm will be able to produce up to 50 kiwis for $1 each and any additional kiwis for $2 each. Assume now that there are two firms are competing a la Bertrand, that is, on prices. To be more precise, each firm maximizes

$$\pi_i(p_i, p_j) = \begin{cases} p_i D(p_i) - C(D(p_i)) & \text{if} \quad p_i < p_j \\ p_i \frac{D(p_i)}{2} - C\left(\frac{D(p_i)}{2}\right) & \text{if} \quad p_i = p_j. \\ 0 & \text{if} \quad p_i > p_j. \end{cases}$$

That is, if the two firms charge the same price, they each sell half the output. We are also assuming that firms have to produce all the quantity that is demanded at a given price, that is, $D(p_i)$, unless they tie, in which case they have to produce $D(p_i)/2$. Show that $p_1^* = p_2^* = 1.5$ is a Nash Equilibrium of this game. (10 points)

16. Compare the quantity and price produced in this equilibrium to the case of perfect competition in the short-run. Comment on the difference between the two cases. Are you surprised? (8 points)

17. (Harder) Find another Nash Equilibrium of this game. (10 points)

18. (Hard) Find all the Nash Equilibria of this game (12 points)

Solution to Problem 4.

1. The demand is a decreasing step-function, as seen in the Figure.

2. The total cost is

$$C(y) =$$

$$C(y) = \begin{cases} y & \text{if} \quad y \leq 10; \\ 10 + 2(y - 10) = 2y - 10 & \text{if} \quad y > 10. \end{cases}$$

The average cost is

$$C(y)/y = \begin{cases} 1 & \text{if} \quad y \leq 10; \\ 2 - 10/y & \text{if} \quad y > 10. \end{cases}$$

The marginal cost is

$$C'(y) = \begin{cases} 1 & \text{if} \quad y < 10; \\ 2 & \text{if} \quad y > 10. \end{cases}$$

(The marginal cost is not defined at $y = 2$, since the function is not differentiable there)

3. The supply function is

$$S(p) = \begin{cases} 0 & \text{if} \quad p < 1; \\ \text{any } y \in \{0, 10\} & \text{if} \quad p = 1; \\ 10 & \text{if} \quad 1 \leq p \leq 2; \\ \text{any } y \in \{10, \infty\} & \text{if} \quad p = 2; \\ y \to \infty & \text{if} \quad p > 2. \end{cases}$$

4. The industry supply function is simply the horizontal addition of the supply functions for the 10 firms:

$$S(p) = \begin{cases} 0 & \text{if} \quad p < 1; \\ \text{any } y \in \{0, 100\} & \text{if} \quad p = 1; \\ 100 & \text{if} \quad 1 \leq p \leq 2; \\ \text{any } y \in \{100, \infty\} & \text{if} \quad p = 2; \\ y \to \infty & \text{if} \quad p > 2. \end{cases}$$

5. Demand equals industry supply for price $p^*_{PC} = 2$ and quantity produced Q^*_{PC} of 200. In reality, any quantity between 110 and 200 equates demand and supply, but we have assumed that consumers consume products whenever indifferent, as in this case they are for units between 110 and 200.

6. Each firm makes \$10 profits, that is, one unit for the first 10 kiwis produced and 0 profit on the additional 10 kiwis. Consumers of type 1 have surplus of \$3 per kiwi, the Consumers of type 2 have surplus of \$1 per kiwi, the Consumers of type 3 have surplus of \$0 per kiwi, as have the Consumers of type 4, since they do not consume. The joint surplus therefore is 10*\$10+100*\$3+10*\$1=\$410.

7. Firms do not make zero profits. This is perfectly consistent with perfect competition in the short-run, that is, with a fixed number of firms. The price is set to equal the marginal cost, so the companies do not earn any profit on the marginal unit, but they can earn profit on the infra-marginal units.

8. In the long-run, given the presence of profits, firms will keep entering until profits are zero. In this case, as long as at least 30 firms enter, they will be able to produce 300 units of production at marginal cost 1, and hence supply it to all of the three hundred consumers. With 30 firms, the supply function is

$$S(p) = \begin{cases} 0 & \text{if} \quad p < 1; \\ \text{any } y \in \{0, 300\} & \text{if} \quad p = 1; \\ 300 & \text{if} \quad 1 \leq p \leq 2; \\ \text{any } y \in \{300, \infty\} & \text{if} \quad p = 2; \\ y \to \infty & \text{if} \quad p > 2. \end{cases}$$

For this function, the equilibrium price p^*_{LR} is 1, and the equilibrium quantity is $Q^*_{LR} = 300$. Similarly to what we said above, any quantity between 201 and 300 equates demand and supply, but we have assumed that consumers consume products whenever indifferent, as in this case they are for units between 201 and 300. More firms can enter than 30, in which case each firm produces less than 10 units; the key thing is that each firm will not produce more than 10 kiwis.

9. In this case, the firm profit is zero. All thirty firms produce 10 kiwis each at a marginal cost of \$1 per kiwi, and they earn \$1 per kiwi. Consumers of type 1 have surplus of \$4 per kiwi, the Consumers of type 2 have surplus of \$2 per kiwi, the Consumers of type 3 have surplus of \$1 per kiwi, and the Consumers of type 4 have surplus of \$0 per kiwi. The joint surplus therefore is \$0+100*\$4+10*\$2+90*\$1+100*\$0=\$510. The surplus therefore is \$100 higher in this case than in the case with no entry. Entry reduces the marginal cost at which firms produce, allowing the creation of additional surplus.

10. The monopolist chooses the profit-maximizing quantity, taking into account costs of production that are the same as in the case of perfect competition with 10 firms. We can best determine this by computing the profit for different prices. Any price $p \leq 1$ yields negative profit, as the firm would sell 300 units produced at an average cost that is higher than 1. All prices $1 < p < 2$ are dominated by $p = 2$, since the company still sells to 200 consumers, and the price $p = 2$ guarantees higher revenue. The profit from $p = 2$ is $200 * 2 - 100 * 1 - 100 * \$2 = \$100$. (the monopolist produces the first 100 units at marginal cost \$1 and the next units at marginal cost \$2). The prices $2 < p < 3$ are dominated by the price $p = 3$, since the company still sells to 110 consumers, and by selling at $p = 3$ it can raise a higher revenue. The profit from $p = 3$ is $110 * \$3 - 100 * \$1 - 10 * \$2 = \$210$ (the monopolist produces the first 100 units at marginal cost \$1 and the next 10 units at marginal cost \$2). The prices The prices $3 < p < 5$ are dominated by the price $p = 5$, since the company still sells to 10 consumers, and by selling at $p = 5$ it can raise a higher revenue. The profit from $p = 5$ is $100 * \$5 - 100 * \$1 = \$400$ (the monopolist produces the first 100 units at marginal cost \$1). The prices above 5 generate 0 profit. Hence, the monopolist maximizes profits by setting $p_M = 5$ and selling $Q_M = 100$.

11. The profit for the monopolist, as we saw. is \$400. The surplus for consumers of Type 1 is \$0, since they pay their willingness to pay, and the surplus for all other consumers is \$0 because they do not purchase. The social surplus, therefore, is \$400, lower than in the case of perfect competition. Monopoly generates deadweight loss because in raising the price to maximize profits, it makes the product so expensive that certain consumers do not buy. In this case, in monopoly only consumers of type 1 purchase – this generates a loss of surplus relative for perfect competition.

12. The monopolist will charge \$5 to consumers of type 1, \$3 to consumers of type 2, and \$2 to consumers of type 3. It will not sell to consumers of type 4, as this would generate negative profits on the margin (those additional units would need to be produced at \$2 per piece). The quantity sold in this case will be 200.

13. The firm profit is $100 * (\$5 - \$1) + 10 * (\$3 - \$2) + 90 * (\$2 - \$2) = \$410$. The consumer surplus is zero, since the consumer is offered exactly its reservation value. Hence, the joint surplus is equal to the firm profit, \$410.

14. The total surplus in monopoly with perfect price discrimination is equal to the surplus in the case of perfect competition in the short-run, and higher than in the case of standard monopoly. The ability to set different prices for different consumers eliminates the deadweight loss.

15. If the two firms set prices equal to 1.5, they will receive demand $D(1.5)$ for 200 units, that is, 100 units per company. They produce these units half at marginal cost of \$1 and half at marginal cost of \$2. The profit of each of the firms, therefore, is $1.5*100 - 50*\$1 - 50*\$2 = \$0$. Firms are making zero profits in equilibrium. We now examine if either firms has an incentive to deviate, that is, if there is another price that yields positive profits. We examine the deviations for firm 1 - given that the game is symmetric, the symmetric reasoning applies to firm 2. A deviation to a price $p < 1.5$ yields negative profits since the firm has to cater to the whole 200 consumers: the profit is $p * 200 - 50 * \$1 - 150 * \$2 = 200p - 350$, which is negative for $p < 7/4$. Any deviation to a price p higher than 1.5 provides zero profit since then the firm is not offering the lowest price. This proves that $p_1^* = p_2^* = 1.5$ is a Nash Equilibrium of this game.

16. By the same token, $p_1^* = p_2^* = p^*$ is an equilibrium for all $1.5 \leq p^* \leq 7/4$. In all of these cases, each firm makes non-negative profits, since $p^* * 100 - 50 * \$1 - 50 * \$2 = 100p^* - 150$ is positive for $p^* \geq 1.5$. Any deviation to a lower price will yield negative profits, by the argument above. Any deviation to a higher price will yield zero profits since then the firm is not offering the lowest price any more. In addition, $p_1^* = p_2^* = p^*$ is an equilibrium for $7/4 \leq p^* \leq 2$. In equilibrium, the firm earns profits $100p^* - 150$. A deviation to a lower price p yields profits $200p - 350$, as we saw above. Consider the most advantageous deviation, which is a deviation to a price that is only slightly below p^*, let's call it $p^* - \varepsilon$. Then, there is no deviation if $100p^* - 150 \geq 200(p^* - \varepsilon) - 350$, or $100p^* \leq 200 + 200\varepsilon \leq 200$. Hence, for $p^* \leq 2$ there is no profitable deviation.

17. We go by step.

    - There is no other equilibrium with $p_1^* = p_2^*$.
        - If $p_1^* = p_2^* < 1.5$, the firm makes negative profits in equilibrium and is better off deviating to a higher price (hence making zero profits).
        - For $3 \geq p_1^* = p_2^* = p^* > 2$, the firms would earn $p^* * 55 - 50 * \$1 - 5 * \$2 = 55p^* - 60$ (the now sell to only 110 consumers, 55 per firm). By deviating to a slightly lower p (but still above 2), let's say $p^* - \varepsilon$, a firm can earn $(p^* - \varepsilon) * 110 - 50 * \$1 - 60 * \$2 = 110(p^* - \varepsilon) - 170$. The deviation is profitable if $110(p^* - \varepsilon) - 170 \geq 55p^* - 60$, or $55p^* \geq 110 + 110\varepsilon$. Hence for $p^* \geq 2$ there is a profitable deviation, provided we make ε small enough.
        - For $5 \geq p_1^* = p_2^* = p^* > 3$, the firms would earn $p^* * 50 - 50 * \$1 = 50p^* - 50$, while by deviating to a lower p (but still above 2), let's say $p^* - \varepsilon$, a firm can earn $(p^* - \varepsilon) * 50 - 50 * \$1 - 50 * \$2 = 100(p^* - \varepsilon) - 150$. The deviation is profitable if $100(p^* - \varepsilon) - 150 \geq 50p^* - 50$, or $50p^* \geq 100 + 100\varepsilon$. Since $p^* \geq 3$, there is a profitable deviation, provided we make ε small enough.
        - A price $p^* > 5$ would never be an equilibrium, as it would garner zero profits.

- Next, consider equilibria with $p_1^* < p_2^*$.
  (a) No equilibria can exist for $1 < p_1^* < 7/4$, since in this case firm 1 would be earning negative profits, and could deviate to a high enough price: the profits would be $p*200-50*\$1-150*\$2 = 200p - 350$, which is negative for $1 < p_1^* < 7/4$.
  (b) For for $p_2^* > p_1^* > 7/4$, no equilibria exists for a different reason: Since firm 1 would be making profits while firm 2 is making 0 profits, firm 2 can lower its price slightly below the price of firm 1 and "steal" the profits of firm 1.
- To sum up, all the Nash equilibria take the form $p_1^* = p_2^* = p^*$ is with $1.5 \leq p^* \leq 2$.